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Methods to convert local sampling coordinates into geographic information system/global positioning systems (GIS/GPS)-compatible coordinate systems

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FIELD NOTE

Methods to Convert Local Sampling Coordinates into Geographic Information System/Global Positioning Systems (GIS/GPS)–Compatible Coordinate Systems

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ABSTRACT

Laying out a sampling transect in the field is a common task when researching natural systems and resources. With widespread availability of global navigation satellite systems (GNSS), such as the US global positioning system (GPS), it is becoming more common to resurvey legacy transects to establish them in globally referenced coordinate systems such as geodetic latitude/longitude or planimetric systems such as the Universal Transverse Mercator (UTM) or the State Plane Coordinate System (SPCS). Transforming local coordinates into a globally referenced coordinate system allows (1) disparate legacy surveys to be combined into a common geographic information system (GIS) database, (2) new GPS measurements to be incorporated into that same database, and (3) GPS-based navigation to be used for plot establishment and resampling. This article presents the mathematics necessary to determine the globally referenced planimetric coordinates of established linear, rectangular, or nominally rectangular transects (such as a rhombus) using formulas that are easily implemented on a spreadsheet. In addition, methods are given to determine the planimetric coordinates of new transects.

Keywords: legacy, transect, sampling, planimetric, coordinate systems

Before the recent popularization of geographic information systems (GIS) and the advent of global navigation satellite systems (GNSS), there was little benefit for fieldworkers to establish plots in a globally referenced coordinate system such as latitude and longitude. In natural resources work, e.g., latitude and longitude (called geographic coordinates) are used mainly for navigational purposes and typically are determined by scaling from the graticule shown on a topographic map. Once an area of interest was reached, a local planimetric coordinate system was established for transect and quadrat plot sampling (Scheaffer et al. 1990, Avery and Burkhart 2002). Hence, most legacy sites used local surveys because they used arbitrary coordinate pairs for the origin and may not have been oriented with any concern for north.

In general, local surveys are incommensurate and therefore can not be meaningfully combined with each other. Transformation of local surveys into a global coordinate system allows valuable historical data to be combined in a GIS with publicly available data sets. Furthermore, use of a global coordinate system allows the use of a handheld global positioning system (GPS) receiver that can aid navigation, plot establishment, and plot resampling. Plots established with a GPS also can be meaningfully combined in a GIS without any modification. The implementation of three GNSSs, namely, the US GPS, the Russian Global'naya Navigatsionnaya Sputnikovaya Sistema (GLONASS), and the European Union Galileo system, provide enough satellite coverage that satellite-based surveying in forests will likely become far more accurate and common in the future (Meyer et al. 2002).

Coordinate Systems

A globally referenced geodetic coordinate system is a coordinate system that is capable of assigning consistent coordinates to anywhere on, above, or within the Earth. Consequently, globally referenced geodetic coordinate systems are spherical in nature (as opposed to planimetric) and intrinsically three dimensional (Meyer et al. 2005, Moritz 2000). By far, the most common geodetic system is latitude and longitude (Meyer 2002). However, few fieldworkers choose to work in spherical coordinates because it is not practicable to fix closely spaced latitudinal and longitudinal coordinates and it is not necessary to consider the curvature of the Earth on typically small-scale plot transects (less than 10 km). Instead, plots typically reside in local planimetric coordinates, which assume the Earth is flat. Although the exact definitions of geodetic latitude and longitude are somewhat subtle (Meyer 2002), for these purposes, the commonplace notions suffice, namely,

Latitude is the angle in the plane of the meridian from the equator to the point of interest, reckoned positive to the north (Figure 1a).

Longitude is the angle in the equatorial plane from the prime meridian to the meridian holding the point of interest, reckoned positive toward the east (Figure 1a).

Planimetric coordinates can be derived rigorously from geodetic latitude and longitude via cartographic projections, such as the Lambert Conformal Conic or the Transverse Mercator. These transformations are invertible, meaning that projected eastings and

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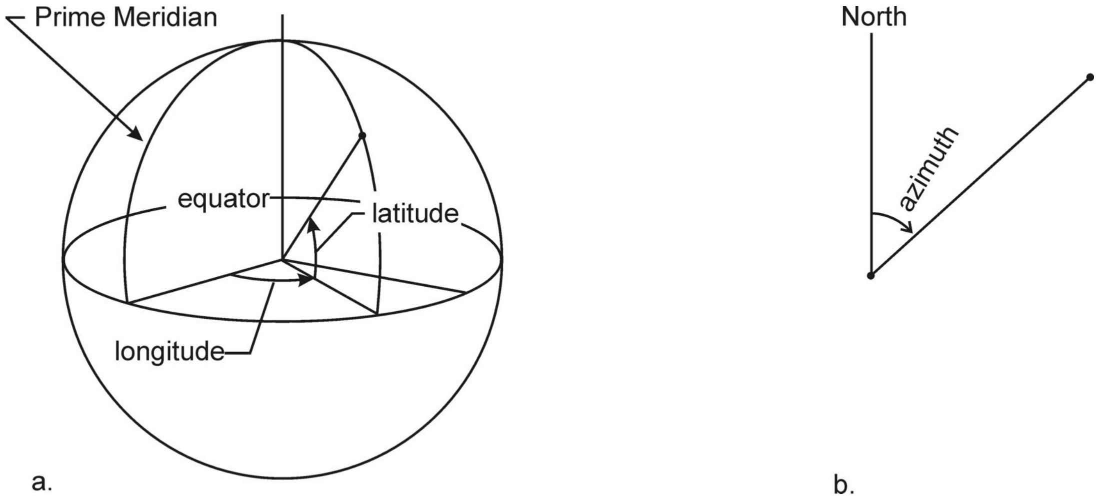

Figure 1. (a) Latitude and longitude are the indicated angles. The point of interest is the solid circle. (b) Azimuth is the angle from the meridian (north) clockwise to the point of interest.

northings unambiguously define a single latitude/longitude pair to which they correspond and vice versa. Thus, there is no important reduction in geospatial rigor when using a globally referenced planimetric coordinate system. The Universal Transverse Mercator (UTM) is an example of a formal, globally referenced planimetric coordinate system.

Geodetic Datums

A geodetic datum (or, simply, "datum") is a mechanism by which geodetic coordinates (latitude and longitude) are assigned to locations on the Earth. Datums are constructed for particular countries or regions. Therefore, to maximize precision, a fieldworker must select the datum appropriate to the area of interest. For example, the North American Datum of 1983 (NAD 83) (Schwarz 1989) is preferred for plot establishment in North America because NAD 83 is the most modern datum for that region (except Mexico) and it is rigorously compatible with the International Terrestrial Reference Frame of 2000 and the World Geodetic System of 1984 (WGS 84), the datum for GPS. If working in Australia, one would use the Geocentric Datum of Australia of 1994 (GDA 94) or the South American Geocentric Reference System (SIRGAS) for determining coordinates in South America. For detailed information on datums we refer the reader to Elithorp and Findorff (2003). If you are unsure of what datum is best for your location, you should consult a geodesist or a land surveyor or contact the National Geodetic Survey.

Projected Planimetric Coordinates

We believe that projected planimetric coordinate systems, such as UTM or State Plane Coordinate System (SPCS), are simpler and more popular than latitude and longitude and assume most fieldworkers would prefer to work in this type of system. For transects so large that the curvature of the Earth can not be considered negligible (approximately more than 10 km in length; Bomford [1980]), this material can be developed in latitude and longitude and would correctly handle the curvature of the Earth, but this is outside the scope of this study.

GPS naturally produces coordinates in globally referenced coordinate systems, such as latitude and longitude or derived planimetric systems, such as the UTM or the SPCS. This article shows how to transform local coordinates into a GPS-compatible coordinate system.

UTM

The UTM coordinate system is very popular for at least two reasons. First, as the name suggests, it is applicable to any location on Earth between 80° south latitude and 84° north latitude, which is not true for other coordinate systems. Second, although UTM has global applicability apart from the poles, it maintains distance error distortions to be, at worst, 1 part in 1,000 by subdividing the Earth into 60 zones starting at the International Date Line, each being 6° of longitude in width. Perhaps the main disadvantage of the UTM coordinate system is also one of its strengths, namely, that it imposes a linear scale distortion of 1:1,000 at the edges and center of the zones. Such distortion is unacceptable for survey-quality mapping but is usually suitable for resource inventories. See Snyder (1987) for full details of the UTM system and common cartographic projections. We will provide concrete examples in the UTM system but everything in this study is equally applicable to any other globally referenced planimetric coordinate system.

SPCS

If your survey requires more precision, the SPCS (Stem 1995) is an alternative to the UTM system and is more precise and favored by land surveyors (linear scale distortion of less than 1 part in 10,000). Although the SPCS is only defined within the United States, most countries have planimetric coordinate systems that are similar in concept. The SPCS subdivides states along political boundaries into inter- and intrastate mapping zones. Zones in which their long dimension is north–south are projected using a Transverse Mercator projection and those with a long east–west dimension are projected using a Lambert Conformal Conic projection; the "panhandle" of Alaska is projected using an Oblique Mercator projection. Although SPCS achieves greater precision by using small zones, its disadvantage is that its smaller zones have local coordinates that are not commensurate across zone boundaries; therefore, SPCS surveys must be fairly small in extent and typically within a single county.

Geodetic Directions

The mathematics that follows depends in part on azimuth. Azimuth is an angle that describes the direction from one place to

Table 1. Easting and northing UTM coordinates for a transect starting at (465236.10 E, 4434167.50 N) length 500 m and oriented 291° north.

i	0	1	2	3	4	5
Easting	365236.10	365142.74	365049.38	364956.03	364862.67	364769.31
Northing	4434167.50	4434203.34	4434239.17	4434275.01	4434310.85	4434346.68

another. Geodetic azimuth, as opposed to magnetic azimuth, uses a meridian (typically north) as its zero reference and is reckoned positive clockwise (Figure 1b). Generally, it is not possible to directly determine geodetic azimuth in the field because magnetic compasses indicate magnetic north, not geodetic north. This can be overcome either by establishing rigorous survey control with an origin monument and an azimuth mark or, more commonly and less accurately, adjusting magnetic north by the local magnetic declination.

Linear Transects

The mathematics in this article will be presented using vector algebra. We denote points and vectors in boldface, e.g., **v**. As coordinate pairs, points and vectors are given as easting and northing values, in that order. Scalar quantities will be given in italics, e.g., *i*.

Suppose it is desired to stake out a transect along a straight line. The length of the transect L, its origin **o**, its azimuth a, and the number of stations (sample plot locations) along the transect (not including the origin) n are given. The solution depends on vector algebra in the following way. The addition of a vector and a point is another point offset from the first in the direction and by a distance equal to those of the vector. Using this fundamental relationship, the location of stations will be given by adding an appropriate vector to the origin. The vector is determined from the transect length and azimuth:

$$\mathbf{v} = (L \sin a, L \cos a) \quad (1)$$

Then, the ith station $\mathbf{s}_i$ is given by

$$\mathbf{s}_i = \mathbf{o} + (i/n)\mathbf{v}. \quad (2)$$

EXAMPLE 1. The origin of a transect is to be located in UTM zone 17 at **o** = (365456.10 E, 44343760567.50 N). The transect is to be 500 m in length, have five stations not including the origin, and oriented at an azimuth of 291°. The vector **v** is given from Equation 1 as

$$\begin{aligned}\mathbf{v} &= (L \sin a, L \cos a)\\ &= (500 \sin 291°, 500 \cos 291°)\\ &= (-466.79, 179.184).\end{aligned}$$

The 0th station is given by $i = 0$ and is the origin of the transect itself. The first station is given from Equation 2 as

$$\begin{aligned}\mathbf{s}_1 &= \mathbf{o} + \tfrac{1}{5}\,\mathbf{v}\\ &= (365236.1, 4434167.5) + 0.2(-466.79, 179.184)\\ &= (365236.1 - 93.358, 4434167.5 + 35.837)\\ &= (365142.74, 4434203.34).\end{aligned}$$

The coordinates for the origin and all five stations are given in Table 1. As a check, the Pythagorean theorem gives the length of the transect as

$$\begin{aligned}L &= \sqrt{(e_0 - e_5)^2 + (n_0 - n_5)^2}\\ &= \sqrt{(365236.10 - 364769.31)^2 + (4434167.50 - 4434346.68)^2}\\ &= \sqrt{-466.79^2 + 179.184^2}\\ &= 500.\end{aligned}$$

Coordinates of a Legacy Transect

Suppose it is desired to determine the UTM coordinates of an existing legacy transect. We assume that the transect's origin and the end point have known planimetric coordinates; perhaps they were determined with a high-accuracy GPS receiver. If the intermediate stations are equispaced or believed to be so, then, Equation 2 is suitable as given and the vector from the origin to the end point can be determined with Equation 1. Otherwise, the horizontal separations between the stations can be measured too, and Eqution 2 can be modified as

$$\mathbf{s}_i = \mathbf{o} + \frac{d_i}{L}\mathbf{v}, \quad (2a)$$

where $d_i = \sum_{j=0}^{i} \delta_j$, $0 \le i < n$ and the δ_j are the horizontal distances (separations) between the sample locations and $\delta_0 \equiv 0$. $L = \sum_{j=0}^{n-1} \delta_j$ is the total horizontal length of the transect, which also can be deduced from the Pythagorean theorem,

$$L = \sqrt{(e_n - e_o)^2 + (n_n - n_o)^2}, \quad (3)$$

and the azimuth is given from

$$a = \arctan(e_n - e_o/n_n - n_o). \quad (4)$$

The formula for the azimuth requires additional explanation. An azimuth can, of course, vary from 0° (north) up to just less than 360°. Therefore, it is critical that the arctangent function be able to discriminate all four quadrants. This requires that the function take two arguments, not the single ratio of the eastings and northings as shown. For example, some spreadsheets support the ATAN2 function, which will function as necessary.

EXAMPLE 2. Given a linear transect origin at UTM zone 14 (101412.43 E, 55213760509.21 N) and a transect end point at (101415.98 E, 55203760596.19 N), find the length of the transect and its azimuth:

$$\begin{aligned}L &= \sqrt{(e_n - e_o)^2 + (n_n - n_o)^2}\\ &= \sqrt{(101615.98 - 101332.43)^2 + (5520696.19 - 5521509.21)^2}\\ &= \sqrt{283.55^2 + (-813.02)^2}\\ &= 861.05,\end{aligned}$$

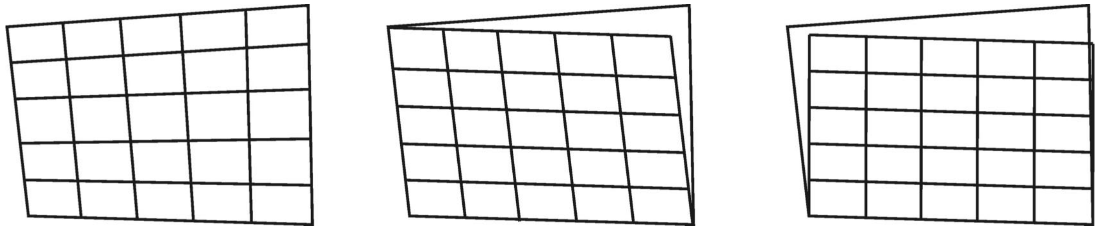

Figure 2. Three ways to subdivide a legacy quadrilateral transect.

$$a = \arctan(101615.98 - 101332.43/5520696.19 - 5521509.21)$$
$$= \arctan(283.55/-813.02)$$
$$= 160.77°.$$

COMMENT. Equation 4 can be used to establish the azimuth between any two points. Therefore, it could be used to determine the azimuth with which to stake out a new transect by locating the transect's origin by navigating to its position with a GPS, and then creating another monument, called an azimuth mark, anywhere visible from the origin and measuring its position with the GPS. Then, use Equation 4 to determine the actual azimuth from the origin to the azimuth mark to calibrate a magnetic compass to geodetic north; or use it as a back site if laying out the transect with a total station.

EXAMPLE 2A. Suppose it is desired to assign UTM coordinates to unequally spaced stations along an existing transect. The origin coordinates were determined to be UTM zone 14 (101412.43 E, 55213760509.21 N) and a corrected compass reading gives the transect azimuth as 15° (N 15 E). A laser rangefinder or a tape and inclinometer was used to determine the horizontal distances separating five sample locations ($n = 5$), namely, 50, 30, 40, and 60 m. Thus, $\delta_0 \equiv 0$, $\delta_1 = 50$, $\delta_2 = 30$, $\delta_3 = 40$, and $\delta_4 = 60$.

The horizontal length of the transect line is the sum of the separation distances, 180 m. From Equation 1, **v** is computed to be $\mathbf{v} = (L \sin \alpha, L \cos \alpha) = 180 \sin 15, 180 \cos 15) = (46.587, 173.867)$. Then,

$$d_0 = \sum_{j=0}^{i} \delta_j = \sum_{j=0}^{0} \delta_j = \delta_0 = 0,$$

$$d_1 = \sum_{j=0}^{1} \delta_j = \delta_0 + \delta_1 = 0 + 50 = 50,$$

and so on up to

$$d_4 = \sum_{j=0}^{4} \delta_j = \delta_0 + \delta_1 + \delta_2 + \delta_3 + \delta_4$$
$$= 0 + 50 + 30 + 40 + 60 = 180.$$

So, from Equation 2a, we compute the coordinates of two sample locations, $\mathbf{s}_0$ and $\mathbf{s}_2$, to illustrate the use of the equation.

$$\mathbf{s}_0 = \mathbf{o} + \frac{d_0}{L}\mathbf{v} = \mathbf{o} + \frac{0}{L}\mathbf{v} = \mathbf{o} = (101332.43, 5521509.21)$$

and

$$\mathbf{s}_2 = \mathbf{o} + \frac{d_2}{L}\mathbf{v}$$
$$= \mathbf{o} + \left(\frac{0 + 50 + 30}{180}\right)\mathbf{v}$$
$$= (101332.43, 5521509.21) + \left(\frac{80}{180}\right)(46.59, 173.87)$$
$$= (101332.43, 5521509.21) + (46.59, 173.87)$$
$$= (101353.14, 5521586.48).$$

Rectangular Transects

Determining the geodetic coordinates of the subplots within nominally rectangular transects can be difficult and tedious. The plot itself often is not actually rectangular, either by design or because of errors in its creation. This gives rise to three different interpretations concerning its internal subplots (Figure 2):

1. Subplots are to evenly subdivide the actual quadrilateral shape as the plot.
2. Subplots are to lay parallel to the local x and y axis defined by the two sides of the plot intersecting at the plot's origin point. This implies subplots are rhomboids and will not, in general, exactly match the farther borders of the plot.
3. Subplots are to be actually rectangular and lay parallel to one side of the plot.

Any one of these arrangements is reasonable; so, we present formulas for each case. In all cases we assume the plot is to have been laid out as a grid of ($n \times m$) rows and columns of subplots.

Quadrilateral Subplots

The problem is, given the geodetic planimetric coordinates of the four points defining a four-sided transect, provide the coordinates of the ($n \times m$) internal subplots such that each side of the transect is subdivided into equal-sized partitions (see Figure 4). We use a mathematical surface, the *Coons patch*, to solve this problem (Coons 1967, Faux and Pratt 1979 p. 198–203). A Coons patch is a *ruled surface*, meaning that it is a surface generated from a family of straight lines. Although Coons patches need not have straight-line boundaries, we will present them in that form because it suits our purpose.

The Coons patch surface is parameterized by two logical variables, u and v. These variables essentially define the x and y directions as defined by the edges of the plot; note that they have no a priori relationship to east or north. Both u and v can have values in the

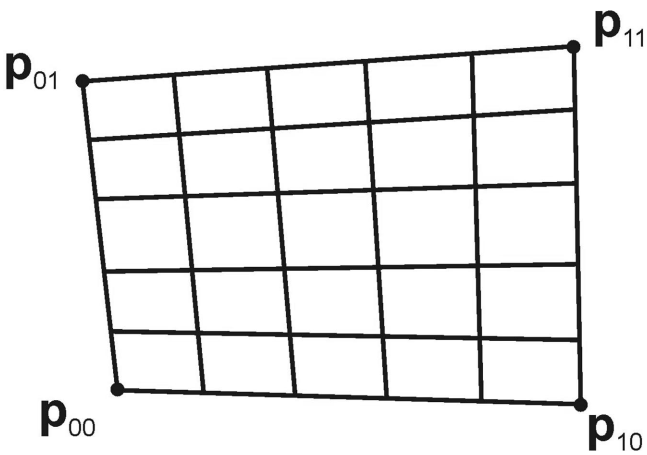

Figure 3. The four corner points of the transect. The origin corresponds to *u* = 0, *v* = 0. The bottom right corner corresponds to *u* = 1, *v* = 0. The top left corner corresponds to *u* = 0, *v* = 1. The top right corner corresponds to *u* = 1, *v* = 1.

range [0, 1]. We denote the Coons patch as $\mathbf{s}(u, v)$, meaning that the function $\mathbf{s}$ evaluated at parameter values u,v yields a point within the patch. The four corner points of the plot are denoted $\mathbf{p}_{00}$, $\mathbf{p}_{01}$, $\mathbf{p}_{10}$, and $\mathbf{p}_{11}$ (Figure 3). With this notion, the formula for a Coons patch is

$$\mathbf{s}(u, v) = (1 - u)(1 - v)\mathbf{p}_{00} + u(1 - v)\mathbf{p}_{10} + (1 - u)v\mathbf{p}_{01} + uv\mathbf{p}_{11} \quad (5)$$

The subscripts of the corner points indicate the u, v pair that correspond to that point, i.e., $\mathbf{s}(0, 0) = \mathbf{p}_{00}$, $\mathbf{s}(0, 1) = \mathbf{p}_{01}$, $\mathbf{s}(1, 0) = \mathbf{p}_{10}$, and $\mathbf{s}(1, 1) = \mathbf{p}_{11}$. For example, let $u = 1$ and $v = 0$. Then

$$\mathbf{s}(1, 0) = (0)(1)\mathbf{p}_{00} + 1(1)\mathbf{p}_{10} + (0)0\mathbf{p}_{01} + 1 \times 0\mathbf{p}_{11} = \mathbf{p}_{10}$$

EXAMPLE 3. Given $\mathbf{p}_{00} = (100, 500)$, $\mathbf{p}_{01} = (80, 620)$, $\mathbf{p}_{10} = (200, 510)$, and $\mathbf{p}_{11} = (210, 630)$, what point is given by $u = 0.3$ and $v = 0.8$? Note that no two sides of this figure are parallel or have the same length.

$$\begin{aligned}
\mathbf{s}(0.3, 0.8) &= (1 - 0.3)(1 - 0.8)\mathbf{p}_{00} + 0.3(1 - 0.8)\mathbf{p}_{10} \\
&\quad + (1 - 0.3)v\mathbf{p}_{01} + 0.3 \times 0.8\mathbf{p}_{11} \\
&= (0.7)(0.2)\mathbf{p}_{00} + 0.3(0.2)\mathbf{p}_{10} + (0.7)0.8\mathbf{p}_{01} \\
&\quad + 0.3 \times 0.8\mathbf{p}_{11} \\
&= 0.14\mathbf{p}_{00} + 0.06\mathbf{p}_{10} + 0.56\mathbf{p}_{01} + 0.24\mathbf{p}_{11} \\
&= 0.14(100, 500) + 0.06(80, 620) \\
&\quad + 0.56(200, 510) + 24(210, 630) \\
&= (0.14 \times 100 + 0.06 \times 80 + 0.56 \times 200 \\
&\quad + 0.24 \times 210,\ 0.14 \times 500 + 0.06 \times 620 \\
&\quad + 0.56 \times 510 + 0.24 \times 630) \\
&= (121.2, 559.0).
\end{aligned}$$

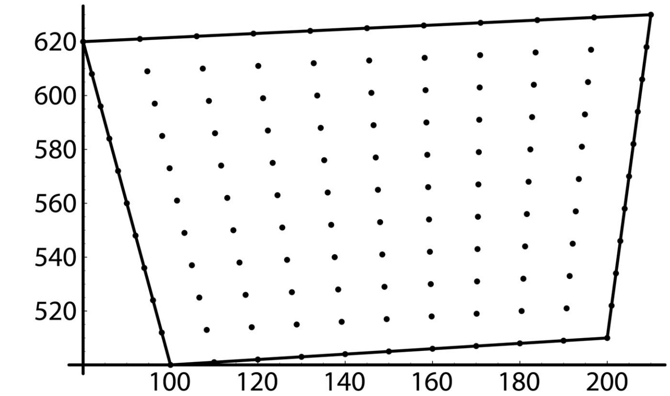

Figure 4. Transect subdivision points given using a Coons patch. Note that all boundaries have been subdivided equally and that the subplots change shape smoothly to interpolate the transect.

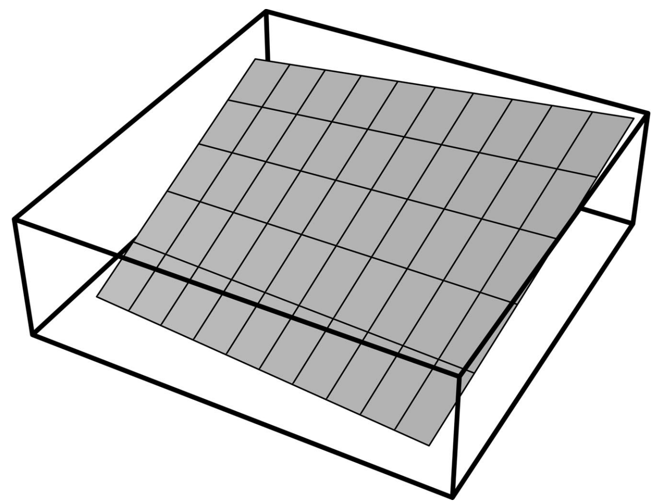

Figure 5. The same surface as shown in Figure 4 but in three dimensions.

Figure 4 shows the transect and all the internal points for u and v varying from 0 to 1 in steps of 0.1 increments. If there are to be n subplots in the u direction and m subplots in the v direction, let u vary in steps of $1/n$ and let v vary in steps of $1/m$.

COMMENT. Equation 5 can be generalized to three dimensions by simply letting the corner points have three coordinates (Figure 5). The rest of the computations follow directly. Figure 5 shows the result if $\mathbf{p}_{00} = (100, 500, 0)$, $\mathbf{p}_{01} = (80, 620, 10)$, $\mathbf{p}_{10} = (200, 510, -20)$, and $\mathbf{p}_{11} = (210, 630, 20)$, plotted against u and v. Note that the surface is twisted but everywhere is composed of straight lines running in the u and v directions.

Rhomboid Subplots

Suppose the subplots are to lay parallel to the "bottom" and "left" axis of the plot. Then, the subplots are rhomboids (Figure 6). We generalize Equation 2 to be

$$\mathbf{s}_{i,j} = \mathbf{o} + \frac{i}{n}\mathbf{v}_u + \frac{j}{m}\mathbf{v}_v, \quad (6)$$

where $\mathbf{s}_{i,j}$ is the ith corner in the u direction and the jth corner in the v direction, $\mathbf{o}$ is the origin point, n is the number of subplots in the u direction, m is the number of subplots in the v direction, and $\mathbf{v}_u$

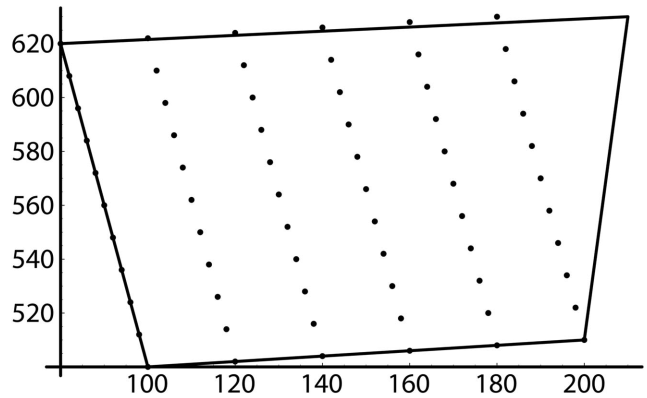

Figure 6. Rhomboid subplots. The subplot is constrained to fall parallel to the "bottom" and "left" axes of the plot. Consequently, some points at the edges fall outside the plot.

and $\mathbf{v}_v$ are vectors aligned with the "bottom" and "left" axis of the plot; $\mathbf{v}_u = \mathbf{p}_{10} - \mathbf{p}_{00}$ and $\mathbf{v}_v = \mathbf{p}_{01} - \mathbf{p}_{00}$.

EXAMPLE 4. Given $\mathbf{p}_{00} = (100, 500)$, $\mathbf{p}_{01} = (80, 620)$, $\mathbf{p}_{10} = (200, 510)$, and $\mathbf{p}_{11} = (210, 630)$ and $n = 5$ and $m = 10$, what point is given by $i = 3$ and $j = 8$?

First,

$$\mathbf{v}_u = \mathbf{p}_{10} - \mathbf{p}_{00} = (200, 510) - (100, 500) = (100, 10).$$

Similarly, $\mathbf{v}_v = (-20, 120)$. Then,

$$\begin{aligned}\mathbf{s}_{i,j} &= \mathbf{o} + \frac{i}{n}\mathbf{v}_u + \frac{j}{m}\mathbf{v}_v \\ &= (100, 500) + 3/5(100, 10) + 8/10(-20, 120) \\ &= (144, 602).\end{aligned}$$

Figure 6 shows all the corners. Notice that the "top" and "right" rows do not match the plot's boundary but that all the subplots honor the axes defined by the "bottom" and "left" plot boundaries.

Rectangular Subplots

To create actually rectangular subplots, $\mathbf{v}_v$ needs to be perpendicular to $\mathbf{v}_u$. Let $\mathbf{v}_u = \mathbf{p}_{10} - \mathbf{p}_{00}$ as before. The condition that $\mathbf{v}_v$ is perpendicular to $\mathbf{v}_u$ is equivalent to insisting that the angle between these vectors be 90° and $\mathbf{b}_u \cdot \mathbf{v}_v = \cos\theta \cdot |\mathbf{v}_u| \cdot |\mathbf{v}_v|$ by the definition of the inner product of two vectors, where $|\cdot|$ denotes a magnitude operator: $|\mathbf{v}| = \sqrt{\mathbf{v}_1^2 + \mathbf{v}_2^2}$, where $\mathbf{v}_1$ denotes the first coordinate of $\mathbf{v}$, and so on. Furthermore, the inner product of two perpendicular vectors equals 0. So, given $\mathbf{v}_u$, find the vector $\mathbf{v}_v$ such that $\mathbf{v}_{u,1} \cdot \mathbf{v}_{v,1} + \mathbf{v}_{u,2} \cdot \mathbf{v}_{v,2} = 0$. This is done easily by choosing $\mathbf{v}_v$ to be either $(\mathbf{v}_{u,2}, -\mathbf{v}_{u,1})$ or $(-\mathbf{v}_{u,2}, \mathbf{v}_{u,1})$. For example, if $\mathbf{v}_u = (100, 10)$, then $\mathbf{v}_v = (-10, 100)$ is perpendicular to $\mathbf{v}_u$: $100 \times 10 + -10 \times 100 = 0$, as required. Then, to lay out rectangular subplots, use Equation 6 as before, having chosen $\mathbf{v}_v$ as described.

Laying Out New Plots

The aforementioned three methods are applicable to this problem by simply using Equations 1 and 2 to establish the coordinates of the plot corners and proceeding as described in the previous section.

Discussion and Conclusions

We have presented techniques with examples to determine the globally referenced planimetric coordinates of established linear, rectangular, or nominally rectangular transects (such as a rhombus). These broadly applicable techniques can enable the one to convert local coordinates from legacy surveys into a globally referenced coordinate system such as UTM. Techniques presented are easily implemented on a spreadsheet. We also include how to determine the planimetric coordinates of new transects.

Because many relationships of interest in natural resources operate on multiple spatial and temporal scales, the ability to establish historical information in a globally referenced system (UTM) allows a researcher or manager to take advantage of the many recent breakthroughs in spatial processing and analysis. Many legacy plots have been maintained for decades, accumulating valuable data that now can be combined with remotely sensed information for validation or understanding large-scale processes over time. With the survey grade accuracy possible with GPS, survey control can be easily brought to remote locations that are unapproachable with traditional surveying methods. In addition, establishing new or legacy plot coordinates with the UTM system allows an efficient resampling or expansion of such surveys.

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