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From Concrete to Abstract: Teaching for Transfer of Learning when Using Manipulatives

Penina Kamina & Nithya Iyer

Abstract

One of the most important uses of manipulatives in a classroom is to aid a learner to make connection from tangible concrete object to its abstraction. In this paper we discuss how teacher educators can foster deeper understanding of how manipulatives facilitate student learning of math concepts by emphasizing the connection between concrete objects and math symbolization with, preservice elementary teachers, the future implementers of knowledge. We provide an example and a model, with specific steps of how teacher educators can effectively demonstrate connections between concrete objects and abstract math concepts.

One of the notable expectations that elementary pre-service teachers' state when they start their mathematics method class is to have a better understanding of the mathematics curriculum. A generic response to the question of what they hope to learn in the course is "I want to learn many ways to make math instruction fun for students by integrating manipulatives." To achieve the elementary pre-service teachers' goal of making math fun and interesting, existing preparation programs offers both theoretical and hands-on approaches to teaching and learning the math content. These approaches incorporate the National Council for Accreditation of Teacher Education (NCATE) standards and the National Council of Teachers of Mathematics (NCTM) principles, standards, and visions. To further elucidate these standards, the philosophies of classical theorists such as Piaget, Brunner, Skinner, Dienes, Brownell, and Vygotsky are discussed in light of their implication to teaching and learning mathematics. Other covered topics in prep programs include lesson planning, becoming a reflective practitioner and professional, as well as the exploration of the mathematics process and content strands using hands-on manipulatives and technology.

Despite the extensive efforts above, we find that elementary pre-service teachers often encounter difficulties transferring knowledge from enactive manipulatives to math symbolization and abstraction. This calls for a need to investigate the issue of transference of knowledge from concrete to abstract when manipulatives are used in mathematics with pre-service teachers.

In this paper, we point out a model for consideration by teacher educators of how to demonstrate to pre-service teachers to strike a balance of making math instruction fun and a worthwhile task, simultaneously First, research regarding manipulatives is reviewed, followed by an example of what pre-service elementary teachers sometimes do. Lastly, a model of how manipulatives can be used to transfer learning from concrete to abstract is discussed with specific steps that teacher educators can implement to encourage pre-service teachers to promote transfer when using manipulatives.

Research Perspectives of Manipulatives

Manipulatives are defined as concrete objects used to help students understand abstract concepts in the domain of mathematics (McNeil & Jarvin, 2007). For decades, researchers have either encouraged or discouraged the use of manipulatives in the classroom. Some acknowledge that manipulatives help students better understand abstract concepts in the domain of mathematics (Sowell, 1989), while others have found them ineffective (Ambrose, 2002; Jarvin, McNeil, & Sternberg, 2006; McNeil, Uttal, Jarvin, & Sternberg, 2007; Resnick & Omanson, 1987; Thompson, 1992).

In contrast to these criticisms, Furner, Yahya and Duffy (2005) suggest that, "the use of manipulatives provides teachers with great potential to use their creativity to do further work on mathematics concepts as an alternative to merely relying on worksheets. Consequently, students are learning mathematics in an enjoyable way, making connections between the concrete and the abstract"(p. 17). McNeil & Jarvin (2007) summarize several benefits of manipulatives as follows: (1) they provide an additional resource in learning mathematics. (2) They help children connect with real-world knowledge, and (3) they help increase memory and understanding.

Despite these benefits, however, manipulatives do not guarantee success if teachers use them primarily for fun and fail to use them effectively. Studies against manipulatives suggest that teachers tend to view manipulative activities as play time (Uttal, Scudder, & DeLoache, 1997; Green, Piel, & Flowers, 2008). For instance, Moyer's (2001) study of 10 middle schools teachers' notes that teachers found the use of manipulatives to be fun and rewarding with students, but they did not see the value of manipulatives as tools for learning math. According to Moyer (2001), the reasons why manipulatives do not work are (i) they are not used effectively in the classroom and (ii) they are poorly perceived. Teachers simply use manipulatives for fun or for adding variety to their teaching, instead of using manipulatives to engage students in mathematics.

Another issue concerning the use of manipulatives is the requirement for dual representation, or understanding manipulatives as both concrete objects, and as symbols of mathematical concepts (Uttal, Scudder, & DeLoache, 1997). Acquisition of dual representation skill calls for additional cognitive resources that are missing in developing children (McNeil & Jarvin, 2007). According to Boulton-Lewis (1998), while children have the ability to manipulate the objects as well as assign them appropriate names, they are unable to identify how the mathematical concept represented by the object corresponds to its tangible symbol.

The above differing research perspectives hold some hints of truth and its key to find a common ground. The NCTM recommends that pre-service teachers "use representation to model and interpret physical, social and mathematical phenomena" (p. 70), where one option is the use of manipulatives in schoolwork. This is significant because "students can represent ideas with objects that can be moved and rearranged. Such concrete representations lay the foundation for the later use of symbols"(p. 137).

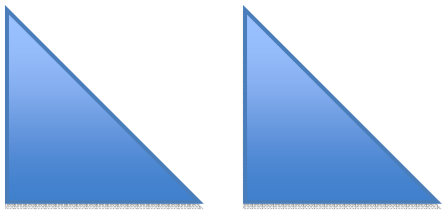
One of the most important uses of manipulatives is to help elementary pre-service teachers make the connection between using manipulatives to facilitate understanding of abstract concepts and procedural knowledge. For such connections to be made, it is helpful to look at manipulatives in the context of transfer of knowledge (Bohan &

Shawaker, 1994), or the ability to apply what is learned in one situation to a different situation (Reed, 1993; Singley & Anderson, 1989; Schunk, 2004; Terwel, van Oers, van Dijk, & van den Eeden, 2009). For transfer to take place, the following conditions must be met: a) a presence of common elements between the topics, and b) learner recognition of common elements (Cox, 1997). Resnick and Ford (1981) suggest that "a more powerful form of instruction "is the use of associationist theory of identical elements where simple concrete tasks assist in transfer of complex learning" (p. 38). This theory suggests that teachers should engage students in the learning process by mediating between the concrete object and the characteristics of the problem situation (Lehtinen and Hannula, 2006). Wookfolk (2008) is also in agreement that, unless prompted or guided, learners fail to apply the problem solving procedures and learning strategies that they have mastered.

Classroom Scenario

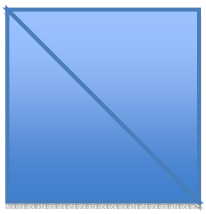
In a typical geometry class session, beyond reviewing basic K – 6 geometrical concepts, elementary pre-service teachers have the chance to explore various activities using tangrams, geoboards, and geometrical computer software to answer application questions, and to subsequently write creative lesson plans. One of the common activity that we assign to elementary pre-service teachers in this topic, is to create different convex polygons using the two small triangular pieces (see Figure 1) out of the tangram set and to write a statement about the area covered by the various polygons constructed after sketching their findings (Activity 1, henceforth).

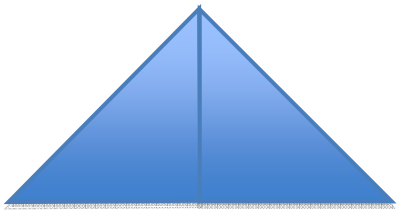
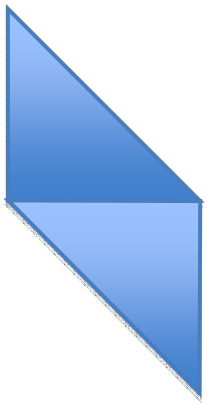
Figure 1



Normally, this is not a daunting task for elementary pre-service teachers. They quickly assemble the triangles together in a number of different ways, discuss the attributes of a convex polygon in small groups, draw sketches of their findings, and write sentence justifications, such as "the areas of a, b and c (Figure 2) are equal because the two triangles used in creating each of the three convex polygons are the same."

Figure 2

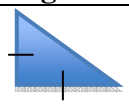
	Some possible elementary pre-service teachers convex sketches to Activity 1
a	

b.	
c.	

While elementary pre-service teachers generally find this Activity 1 fun to do, the extensive whole-class mathematical discussions and the subsequent detailed journaling e.g. congruent lengths, congruent angles, accurate mathematical labeling of the sketches, etc. that ensues, is seen as less enjoyable, due to lack of connection. The above sketches in Figure 2, as is, are where majority of the elementary students are comfortable with. Proceeding on with "minor" details is seen as a drudge, yet it is the core of the subject. Significant mathematical symbolizations and abstractions embedded in the manipulatives are easily pushed aside that should be capitalized on by these future teachers. We believe this is where the teacher educators need to be more assertive to bridge the link between fun and the unpalatable math content. Helterbran (2008) notes that teachers tend to teach as they were taught thus the need for appropriate role modeling.

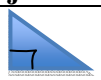
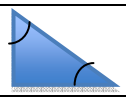
In Activity 1, to show congruence symbolically of the two equal lengths as seen concretely by lining up the two triangles side by side the use of tick marks is employed. A student tangibly requires the two triangles to prove this, but one paper, only one triangle is used (see Figure 3). As seen here, at times in mathematics the elements (Cox, 1997) involved are not exactly identical, therefore we find that processing and harnessing this information from concrete to abstract by an expert is important.

Figure 3

Drawn Triangle with notation	Symbolic meaning
	Same number of tick marks implies congruency (same lengths). Mathematicians interpret that the two sides are congruent if one side has one, two or three tick marks on it and another side has one, two or three, respectively.

Other mathematically correct labeling arising from Activity 1, include a right angle, which is denoted by a little square drawn on the 90° angle (Figure 4a), or curved lines that represent congruent angles (Figure 4b).

Figure 4

	Drawn Object with notation	Symbolic meaning
a.		little square = Right angle; 90^0
b.		curved lines = Congruent (same) angles

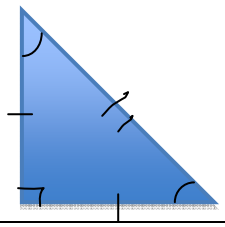
Bridging Concrete to Abstract

What can teacher educators do to link the disconnection between the concrete and the abstract? In this section, we share what we often attempt to do with manipulatives in math methods given our experience with Activity 1 with the pre-service elementary teachers, and why we value this model.

At the beginning of the course we use instructional time by making explicit connections between manipulatives and abstract math concepts to establish a socio-mathematical routine to help elementary pre-service teachers understand mathematics and promote transfer of learning. This approach has three steps to it namely, scaffolding, exploration, and abstraction, which we use with various math concepts and differing manipulatives, from the start of the term until it becomes a classroom norm.

Step 1: Through either direct instruction or scaffolding (Jordan, Schwartz, & McGhie-Richmond, 2009; Olson & Truxaw, 2009), we explore the mathematical attributes of the manipulative relative to abstract math concept being taught. For example, in assigning Activity 1 above, point out the attributes of this triangle (Figure 1), such as the two equal lengths of the sides of this triangle, and discuss its name as an *isosceles right* triangle by displaying the manipulative (enactive), drawing its shape on the board (iconic), pointing out congruent sides, right angle, and acute angles, and mathematically labeling the angle and sides (symbols), as shown in Figure 5 below.

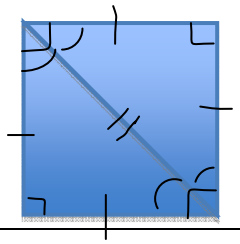
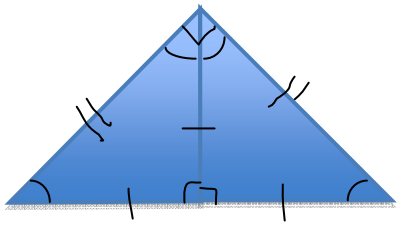
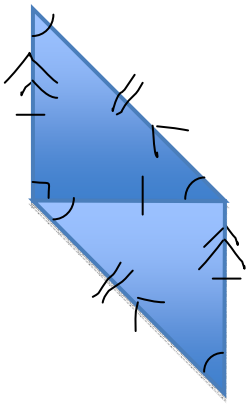
Figure 5

Drawing of the manipulative with symbolic notation


Step 2 is the exploration. We assign an open-ended task and allow the pre-service elementary teachers to come up with different strategies (Bonotto, 2005; Bot, Gossiaux, Rauch, & Tabiou, 2005; Knewstubb & Bond, 2009). Upon completion of the task, we give them a chance to discuss. In this talk we are able to distinguish those pre-service elementary teachers who just stayed with the modeling and demonstration from those who exceeded this by transferring knowledge and applying it to the task.

Step 3 is the abstraction. Require all the elementary pre-service teachers to excel by moving beyond modeling, being proactive, reflective, and writing about all relevant math aspects (Goldstone & Son, 2005; Loughran, 2009). For example, in Activity 1, we require all elementary pre-service teachers to label appropriately (Figure 6) and be exemplary in their explanations.

Figure 6

	Some possible sketches drawn appropriately
a.	
b.	
c.	

We expect elementary pre-service teachers to point out that, despite having polygons in Figure 6a (square), 6b (isosceles triangle) and 6c (parallelogram) possess equal areas, the resulting formed convex polygons are different by (i) comparing and

contrasting the number of sides and angles, and (ii) pointing out which of these attributes are congruent.

Note that the arrow symbolization used in the parallelogram in Figure 6c above, the notation for parallel sides was not discussed in Step 1, since this is the type of skill elementary pre-service teachers should garner by exceeding the given scaffold through the transfer of knowledge from one area to another. With an understanding of the use of tick marks for congruent sides, elementary pre-service teachers should ask themselves if the parallel sides require a different representation or the same. Such discussion elicits the use of one, two, or three arrows to denote parallel lines. Besides the convex polygon sketches, the completion of Activity 1 should also include the use of a *little square* to represent a right angle or 90°, the use of *tick marks* for congruent sides, the use of *curved lines* for congruent angles, and referring to the side of a triangle opposite the right angle as the *hypotenuse*, the angle less than 90° as an *acute angle,* the angle between 90° and 180° as *obtuse,* and the angle between 180° and 360° as *reflexive*.

In conclusion, by emphasizing the effective use of manipulatives, teacher educators can explicitly connect abstract math concepts and manipulatives to establish a socio-mathematical routine to help elementary pre-service teachers understand mathematics and promote transfer of learning. Teacher educators have the responsibility to teach mathematics in such a way that elementary pre-service teachers have a deep understanding of its patterns, function and meaning (NCTM, 2000). As teacher educators, we want to provide meaningful learning experiences for our pre-service teachers in hopes of providing all schools with quality math teachers.

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