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## **Abstract**

This study compares the procurement cost-minimizing and productive efficiency performance of the auction mechanism used by independent system operators (ISOs) in wholesale electricity auction markets in the U.S. with that of a proposed alternative. The current practice allocates energy contracts as if the auction featured a discriminatory final payment method when, in fact, the markets are uniform price auctions. The proposed alternative explicitly accounts for the market clearing price during the allocation phase. We find that the proposed alternative largely outperforms the current practice on the basis of procurement costs in the context of simple auction markets featuring both day-ahead and real-time auctions and that the procurement cost advantage of the alternative is complete when we simulate the effects of increased competition. We also find that a trade-off between the objectives of procurement cost minimization and productive efficiency emerges in our simple auction markets and persists in the face of increased competition.

**Journal of Economic Literature Classification:** C72, D44, L10, L94

**Keywords:** strategic behavior, multi-unit auction, wholesale electricity, Bertrand competition

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# 1 Introduction

The 1990s featured a wave of deregulation and privatization of the electricity industries of several nations including the U.S., U.K., Spain, and Norway. Enhancing competition in the production and sale of wholesale electricity has since become focal in policy discussions related to electricity in the U.S. and abroad (von der Fehr and Harbord (1998), Hunt (2002)). Despite the promises of the increased rigors of market discipline, there remains a great concern that deregulation and privatization of the electricity industry has yet to deliver lower prices to the largest electricity consumers in the U.S. (Apt (2005)). The empirical literature on wholesale electricity markets in California (Borenstein and Bushnell (1999), Joskow and Kahn (2001), Borenstein et al. (2002), Wolak (2003), and Puller (2005)), Texas (Hortaçsu and Puller (2005)), and England and Wales (Green and Newbery (1992), von der Fehr and Harbord (1993), Wolfram (1998, 1999), and Wolak and Patrick (2001)) provides extensive evidence on the existence of generators' unilateral market power and resultant pricing above marginal cost. Further, some studies find evidence of capacity withholding on the part of generators to maintain supracompetitive prices (Joskow and Kahn (2001) and Wolak and Patrick (2001)). Such bad news has placed a premium on finding ways to mitigate market power and increase competition in wholesale electricity markets.

The objective of this paper is to compare the procurement cost-minimizing and productive efficiency performance of two mechanisms for the allocation of energy contracts in U.S. wholesale electricity markets. In the U.S., wholesale electricity markets are organized as multi-unit uniform price auctions run daily by independent system operators (ISOs). Further, these auctions feature dual settlement systems: ISOs oversee both *day-ahead auctions*, run daily for each hour of the following day, as well as *real-time auctions*, run every five minutes during the day. Generators participate in these auctions by submitting offer curves consisting of generation levels and energy prices as well as start-up costs, no-load costs, minimum up and down times, and other technical constraints and costs. The ISO collects these offers from participating generators and combines them with energy bids from load serving entities to construct the aggregate supply and demand curves it uses to clear the market in a cost-minimizing way. The auction mechanism at work in these markets has two components, one pertaining to the method of final payment and one pertaining to the allocation of energy contracts among generators. In the U.S., wholesale electricity markets are uniform price auc-

tions, which means that generators awarded energy contracts are *not* paid their offer price as would be the case in a discriminatory auction, but are paid the market clearing price for the auction. In this paper, we take this final payment method as given and focus on two alternative means of allocating energy contracts among generators.

ISOs clear the market and allocate energy contracts among participating generators in a cost-minimizing way by using a centralized optimization algorithm. *Offer cost minimization*, the so-called “unit commitment problem” in the power systems engineering literature, is the current practice among many ISOs in the U.S. Under offer cost minimization, an ISO selects energy offers from generators by choosing that allocation that minimizes the amount that would be paid to generators were they to be paid their offered prices for their energy despite the fact that these generators will be paid the market clearing price associated with the selected allocation. That is, the problem the ISO solves under offer cost minimization treats the auction’s final payment mechanism as if it were that of a discriminatory auction when in fact it is that of a uniform price auction.

*Payment cost minimization* is an alternative objective for an ISO’s centralized optimization as first noted by Hao et al. (1998). Under payment cost minimization, ISOs award energy contracts to generators so as to minimize the cost of procuring that energy. That is, the problem the ISO solves under payment cost minimization explicitly accounts for the market clearing price associated with the selected allocation. Yan and Stern (2002) provide a formulation of the payment cost minimization problem and highlight the inconsistency between ISOs’ current practice of offer cost minimization and the wholesale auctions’ uniform price final payment method by providing a simple numerical example showing that payment cost minimization returns relatively lower procurement costs to the ISO than does offer cost minimization for a given set of energy offers. More recently, Luh et al. (2005a) develops a solution method for the ISO’s payment cost minimization problem for simplified wholesale electricity auction markets.<sup>1</sup> Numerical testing results show

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<sup>1</sup>Luh et al. (2005a) develop an algorithm that ISOs can implement in uniform price wholesale electricity auctions markets featuring fixed demand, full compensation of generators’ start-up costs, and minimal technical constraints. Luh et al. (2005b) extends the work of Luh et al. (2005a) to the problems caused by demand bids (double auctions) and partial compensation of start-up costs. Numerical testing results show procurement costs that are even lower than those reported in Luh et al. (2005a) for a given set of energy offers.

that payment cost minimization returns procurement costs to the ISO that are relatively lower than those obtained under offer cost minimization for a given set of energy offers.

In this paper, we study the procurement cost-minimization and productive efficiency performance of both offer cost minimization, and its proposed alternative, payment cost minimization. We select these as criteria for comparison of offer cost minimization and payment cost minimization as they are standard performance measures in the economics literature on auctions, though what these researchers typically compare is the performance of alternative final payment methods in auctions (Kahn et al. (2001), Ausubel and Cramton (2002), and Fabra et al. (2004)).<sup>2</sup> Further, procurement costs are focal since the main objective of the Federal Energy Regulatory Commission (FERC), the main body overseeing the implementation of deregulated electricity markets in the U.S., is “to achieve wholesale electricity markets that produce just and reasonable prices and work for customers” (Federal Energy Regulatory Commission (2003, p. 1)).

In particular, we examine the performance of these two formulations in the context of small markets. Focusing the analysis on small markets makes sense for a number of reasons: 1. we can obtain equilibrium results for an entire class of markets; 2. we can simulate the effects of increased competition in the context of small markets by analyzing Bertrand competition; 3. equilibrium analysis with a large number of suppliers is likely intractable.

There are a number of comparisons of the offer cost minimization and payment cost minimization formulations in the power systems engineering literature. Alonso et al. (1999) examine the procurement cost and productive efficiency properties of the two formulations in the context of a simple wholesale electricity market and find that payment cost minimization provides lower procurement costs while offer cost minimization obtains greater productive efficiency. Vázquez et al. (2002) acknowledge that payment cost minimization achieves lower procurement costs, but show that this may be at the expense of distorted incentives for generators’ capacity installation in the long-run, which may translate into higher energy prices in the long-run. As previously mentioned, numerical testing results found in Hao et al. (1998), Yan and Stern (2002), and Luh et al. (2005a, 2005b) show that pay-

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<sup>2</sup>In sale auctions, revenue maximization and consumption efficiency, allocation of the sold item(s) to the highest valuing bidder(s) are analogues to procurement cost minimization and productive efficiency, procurement from the least cost producer(s), in procurement auctions.

ment cost minimization returns lower procurement costs to the ISO than does offer cost minimization, the current practice. The problem with this line of research is that it fails to consider the effects of generators' strategic behavior on the performance of these allocation mechanisms. In this body of research, the wholesale electricity markets being modeled are assumed to be perfectly competitive and generators' offer strategies do not change with changes in the ISO's procurement objective. In this regard, the novelty of this paper is to compare the procurement cost-minimizing and productive efficiency performance of these two procurement algorithms while accounting for the strategic behavior of generators in wholesale electricity auction markets.

The economics literature contains a number of studies concerning wholesale electricity markets. Green and Newbery (1992) construct and analyze symmetric and asymmetric duopoly models of wholesale electricity auctions in England and Wales using the supply function equilibrium framework of Klemperer and Meyer (1989) where generators are assumed to submit continuously differentiable offer schedules to a multi-unit uniform price auction; start-up costs are ignored in the analysis. von der Fehr and Harbord (1993) follow shortly thereafter with an auction theoretic duopoly model of the England and Wales markets. Generators are restricted to submit step function offer curves to a multi-unit uniform price auction. The key result from both Green and Newbery (1992) and von der Fehr and Harbord (1993) is that generators' exercise of unilateral market power leads to supracompetitive pricing in equilibrium. Le Coq (2002) develops and analyzes a two-stage game in which generators precommit to capacity levels and then participate in a multi-unit uniform price auction and finds equilibria in which at least one generator withholds capacity to raise prices to some maximum allowable level. Spear (2003) features a general equilibrium model of imperfect competition in deregulated electricity markets and finds generators' unilateral market power to explain supracompetitive pricing in both off-peak and peak demand periods, price spikes during peak demand periods, and capacity reduction to increase prices in some cases. Fabra et al. (2004) model both discriminatory and uniform price wholesale electricity auctions where generators submit step function offer curves and show that one cannot unambiguously rank the two final payment methods in terms of both costs to consumers and productive efficiency.

While the above research is explicit in its consideration of the effects of generators' strategic behavior in wholesale electricity markets, it fails to

look beyond the auction’s final payment method to consider the other component of the mechanism, the ISO’s method of allocating energy contracts. Knoblauch (2005) is a recent exception in this regard. Knoblauch (2005) features a game theoretic analysis of the procurement cost-minimizing performance of offer cost minimization versus payment cost minimization in the context of simple single-market auction models and finds a case where payment cost minimization delivers *higher* procurement costs to the ISO in the equilibrium of a two-supplier auction, a case which disappears with increased competition so that payment cost minimization always ties or outperforms offer cost minimization on the basis of procurement costs. This paper expands upon the work of Knoblauch (2005) by extending the analysis to a dual settlement system featuring both day-ahead and real-time auction markets and considers both the procurement cost-minimizing and productive efficiency performance of offer cost minimization and payment cost minimization. Further, we enrich generators’ strategy space by examining capacity withholding behavior in tandem with offer pricing behavior.

The remainder of the paper is organized as follows. Section 2 features a simple numerical example to motivate our study of the performance of the two allocation mechanisms in wholesale electricity markets. Here, we find the counterintuitive result that the proposed mechanism, payment cost minimization, delivers *higher* procurement costs to the ISO than does offer cost minimization. Section 3 generalizes the example of Section 2 and shows that this result is not the norm in the context of simple two-supplier auction markets. We also analyze the productive efficiency of the allocations that the two formulations obtain and find a tradeoff between the objectives of payment cost minimization and productive efficiency. In Section 4, we develop a three-supplier model featuring Bertrand competition between the two symmetric suppliers to simulate the effects of increased competition in wholesale electricity markets. We find that payment cost minimization always ties or outperforms offer cost minimization on the basis of procurement costs, but that the tradeoff between the goals of payment cost minimization and productive efficiency persists. Section 5 concludes.



## 2 Example: Simple Wholesale Electricity Auctions, Day-ahead and Real-time

Consider a wholesale electricity market with two suppliers and one ISO. Further, suppose that it features a dual settlement system; that is, this wholesale electricity market features both a day-ahead and real-time market for electricity. Supplier 1 can supply 0, 1, or 2 units of energy, has start-up costs of zero, and selects between two offer prices, offer low ( $O_l$ ) and offer high ( $O_h$ );  $O_l(1) = O_l(2) = 35$  dollars per unit and  $O_h(1) = O_h(2) = 45$  dollars per unit. Further, Supplier 1 may choose whether or not to withhold one unit of its energy capacity from the day-ahead market for exclusive use in the real-time market. Supplier 2, on the other hand, can supply 0 or 1 units of energy, has start-up costs of 15 dollars, and has one offer price,  $O_2(1) = 25$  dollars per unit. Supplier 2 always offers its unit of capacity for sale in the day-ahead market. The ISO has an inelastic demand for 2 units of electricity in the day-ahead market and has an inelastic demand for 1 unit with probability 0.5 or for 0 units of electricity in the real-time market with probability 0.5. Finally, any energy offers that are not selected to run in the day-ahead market “roll over” to the real-time market without adjustment to their offer prices.<sup>3</sup>

We analyze this example in the context of two games. In Example Game 1, the ISO uses the offer cost minimization formulation to select which energy offers to award contracts. In Example Game 2, the ISO uses the payment cost minimization formulation. Since Supplier 1 is the only player with more than one strategy, Example Games 1 and 2 are both one-player games, so we seek out Supplier 1’s equilibrium strategy.

*Example Game 1: Offer Cost Minimization Formulation.*

This is a game of imperfect information, as Supplier 1 is uncertain of whether or not there will be a real-time market when making its withholding and offer pricing decisions. A Supplier 1 strategy is a complete plan of action specifying a Supplier 1 action (withholding and offer pricing) at each of its information sets; there are thirty two Supplier 1 strategies. To simplify the

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<sup>3</sup>This assumption is for mathematical convenience only. It is easy to show that relaxing it does not affect the results. For this reason, we retain it for the remainder of the paper.

exposition, however, we perform backward induction to eliminate the two singleton information sets (information sets  $d$  and  $e$  in Figures 1 and 2) and consider the eight Supplier 1 strategies that remain to compute (subgame perfect) equilibria for the game.

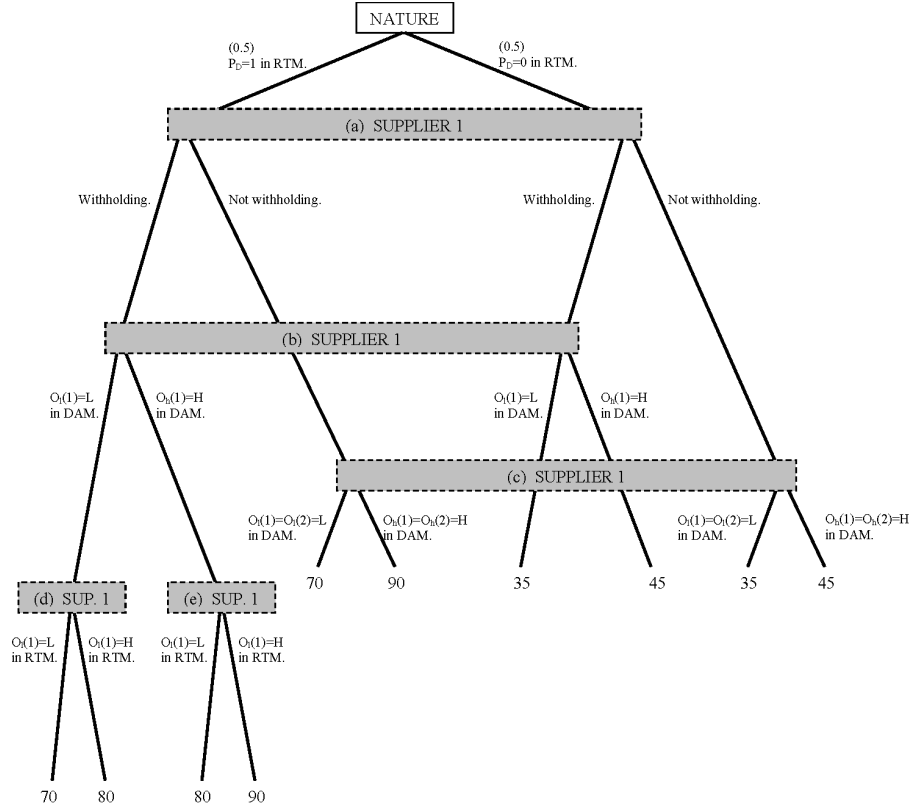


Figure 1: Example Game 1 in Extensive Form

Consider the quintuple of (withholding decision, day-ahead offer price, day-ahead offer price, real-time offer price, real-time offer price) for Supplier 1. If Supplier 1 submits (not withholding,  $O_l$ ,  $O_l$ ,  $O_h$ ,  $O_h$ ), the ISO calculates its expected procurement cost as  $\min\{35 + 15 + 25, 2(35)\} + (0.5)\min\{15 + 25\} = 90$  and Supplier 1's expected payoff is 70. If Supplier 1 submits (not withholding,  $O_h$ ,  $O_l$ ,  $O_h$ ,  $O_h$ ), the ISO calculates its expected procurement cost as  $\min\{35 + 15 + 25, 2(35)\} + (0.5)\min\{15 + 25\} = 90$  and Supplier 1's ex-

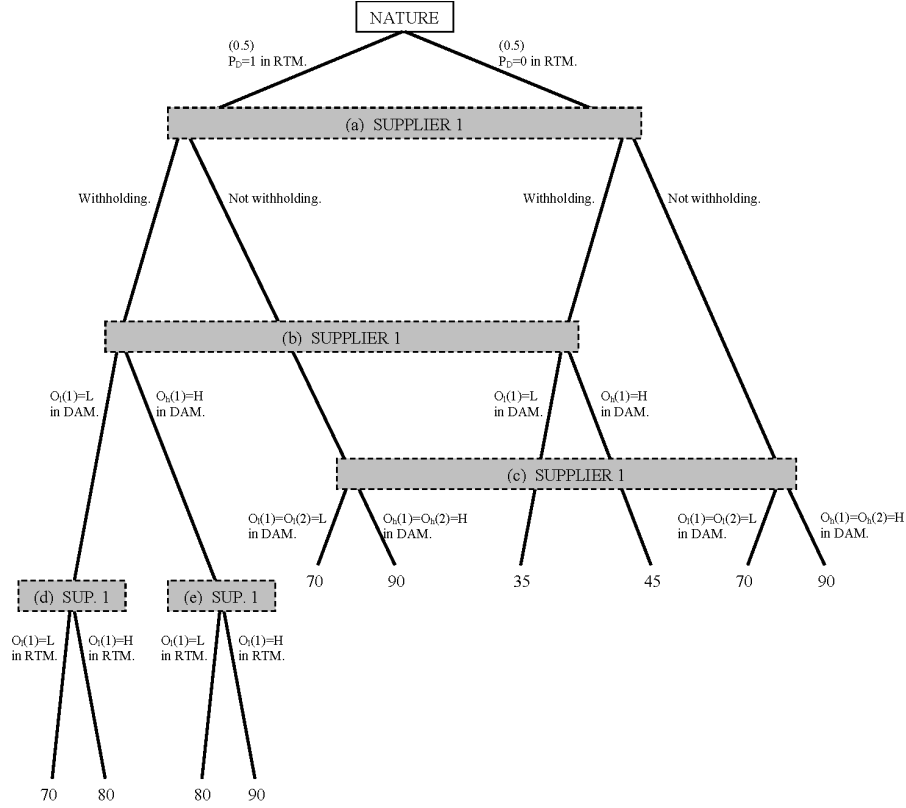


Figure 2: Example Game 2 in Extensive Form

pected payoff is 70. If Supplier 1 submits (not withholding,  $O_l, O_h, O_h, O_h$ ), the ISO calculates its expected procurement cost as  $\min\{45+15+25, 2(45)\} + (0.5)\min\{45\} = 107.5$  and Supplier 1's expected payoff is 67.5. If Supplier 1 submits (not withholding,  $O_h, O_h, O_h, O_h$ ), the ISO calculates its expected procurement cost as  $\min\{45+15+25, 2(45)\} + (0.5)\min\{45\} = 107.5$  and Supplier 1's expected payoff is 67.5. If Supplier 1 submits (withholding,  $O_l, O_l, O_h, O_h$ ), the ISO calculates its expected procurement cost as  $\min\{35+15+25\} + (0.5)\min\{45\} = 97.5$  and Supplier 1's expected payoff is 57.5. If Supplier 1 submits (withholding,  $O_h, O_l, O_h, O_h$ ), the ISO calculates its expected procurement cost as  $\min\{45+15+25\} + (0.5)\min\{45\} = 107.5$  and Supplier 1's expected payoff is 67.5. If Supplier 1 submits (withhold-

ing,  $O_l, O_h, O_h, O_h$ ), the ISO calculates its expected procurement cost as  $\min\{35 + 15 + 25\} + (0.5)\min\{45\} = 97.5$  and Supplier 1's expected payoff is 57.5. If Supplier 1 submits (withholding,  $O_h, O_h, O_h, O_h$ ), the ISO calculates its expected procurement cost as  $\min\{45 + 15 + 25\} + (0.5)\min\{45\} = 107.5$  and Supplier 1's expected payoff is 67.5. Therefore, (not withholding,  $O_l, O_l, O_h, O_h$ ) and (not withholding,  $O_h, O_l, O_h, O_h$ ) constitute equilibria for Example Game 1; the day-ahead market clearing price (MCP) is 35, the real-time MCP is 25, and the ISO's expected procurement cost is 90 in equilibrium.

*Example Game 2: Payment Cost Minimization Formulation.*

This game closely follows Example Game 1, with the exception that now the ISO's optimization explicitly accounts for the MCP that would prevail if certain sets of offers were selected to supply energy. We will proceed as above by considering each of the eight Supplier 1 strategies to compute equilibria for the game.

If Supplier 1 submits (not withholding,  $O_l, O_l, O_h, O_h$ ), the ISO calculates its expected procurement cost as  $\min\{15 + 2\max\{35, 25\}, 2(35)\} + (0.5)\min\{15 + 25\} = 90$  and Supplier 1's expected payoff is 70. If Supplier 1 submits (not withholding,  $O_h, O_l, O_h, O_h$ ), the ISO calculates its expected procurement cost as  $\min\{15 + 2\max\{35, 25\}, 2(35)\} + (0.5)\min\{15 + 25\} = 90$  and Supplier 1's expected payoff is 70. If Supplier 1 submits (not withholding,  $O_l, O_h, O_h, O_h$ ), the ISO calculates its expected procurement cost as  $\min\{15 + 2\max\{45, 25\}, 2(45)\} + (0.5)\min\{15 + 25\} = 110$  and Supplier 1's expected payoff is 90. If Supplier 1 submits (not withholding,  $O_h, O_h, O_h, O_h$ ), the ISO calculates its expected procurement cost as  $\min\{15 + 2\max\{45, 25\}, 2(45)\} + (0.5)\min\{15 + 25\} = 110$  and Supplier 1's expected payoff is 90. If Supplier 1 submits (withholding,  $O_l, O_l, O_h, O_h$ ), the ISO calculates its expected procurement cost as  $\min\{15 + 2\max\{35, 25\}\} + (0.5)\min\{45\} = 107.5$  and Supplier 1's expected payoff is 57.5. If Supplier 1 submits (withholding,  $O_h, O_l, O_h, O_h$ ), the ISO calculates its expected procurement cost as  $\min\{15 + 2\max\{45, 25\}\} + (0.5)\min\{45\} = 127.5$  and Supplier 1's expected payoff is 67.5. If Supplier 1 submits (withholding,  $O_l, O_h, O_h, O_h$ ), the ISO calculates its expected procurement cost as  $\min\{15 + 2\max\{35, 25\}\} + (0.5)\min\{45\} = 107.5$  and Supplier 1's expected payoff is 57.5. If Supplier 1 submits (withhold-

ing,  $O_h, O_h, O_h, O_h$ ), the ISO calculates its expected procurement cost as  $\min\{15 + 2\max\{45, 25\}\} + (0.5)\min\{45\} = 127.5$  and Supplier 1's expected payoff is 67.5. Therefore, (not withholding,  $O_l, O_h, O_h, O_h$ ) and (not withholding,  $O_h, O_h, O_h, O_h$ ) constitute equilibria for Example Game 2; the day-ahead MCP is 45, the real-time MCP is 25, and the ISO's expected procurement cost is 110 in equilibrium.

The equilibria of Example Games 1 and 2 produce a counterintuitive result. Comparing the equilibria under the offer cost minimization formulation to those under the payment cost minimization formulation, we see that the ISO's procurement optimization using the *correct* objective function—that is, the one in line with the auctions' uniform price settlement rule—actually returns *higher* procurement costs in equilibrium. In the section which follows, we show that the results of this example are an anomaly. Further, in Section 4, we demonstrate that no analogue to the results of this example exists when we add Bertrand competition to the model.

### 3 Two-Supplier Wholesale Electricity Auctions

#### 3.1 Procurement Costs

We further explore simple two-supplier auctions of the type found in the example from the previous section. In particular, we will examine to what extent the results of Example Games 1 and 2 are typical. Games 1 and 2 below are identical to Example Games 1 and 2, respectively, except that for Supplier 1,  $O_l(1) = O_l(2) = L > 0$  and  $O_h(1) = O_h(2) = H > L$ ; for Supplier 2,  $O_2(1) = A > 0$  and start-up costs are  $S > 0$ ; and, in the real-time market, the ISO has an inelastic demand for 1 unit of electricity with probability  $\alpha \in (0, 1)$  or for 0 units with probability  $1 - \alpha$ .

We again note that these are games of imperfect information, as Supplier 1 is uncertain of whether or not there will be a real-time market when making its withholding and offer pricing decisions. A Supplier 1 strategy is a complete plan of action specifying a Supplier 1 action (withholding and offer pricing) at each of its information sets; there are thirty two Supplier 1 strategies. To simplify the exposition, however, we perform backward induction to eliminate the two singleton information sets (information sets  $d$  and  $e$  as in Figures 1 and 2) and consider the eight Supplier 1 strategies that remain to compute equilibria for the games. The solutions to Games 1 and 2 break naturally

into four cases.

**Case 1.**  $S + A < L$ .

*Game 1: Offer Cost Minimization Formulation.*

If Supplier 1 submits (not withholding,  $O_l, O_l, O_h, O_h$ ), the ISO calculates its expected procurement cost as  $\min\{L + S + A, 2L\} + \alpha \min\{L\} = (1 + \alpha)L + S + A$  and Supplier 1's expected payoff is  $\pi_1(\text{not withholding, } O_l, O_l, O_h, O_h) = (1 + \alpha)L$ . If Supplier 1 submits (not withholding,  $O_h, O_l, O_h, O_h$ ), the ISO calculates its expected procurement cost as  $\min\{L + S + A, 2L\} + \alpha \min\{L\} = (1 + \alpha)L + S + A$  and Supplier 1's expected payoff is  $\pi_1(\text{not withholding, } O_h, O_l, O_h, O_h) = (1 + \alpha)L$ . If Supplier 1 submits (not withholding,  $O_l, O_h, O_h, O_h$ ), the ISO calculates its expected procurement cost as  $\min\{H + S + A, 2H\} + \alpha \min\{H\} = (1 + \alpha)H + S + A$  and Supplier 1's expected payoff is  $\pi_1(\text{not withholding, } O_l, O_h, O_h, O_h) = (1 + \alpha)H$ . If Supplier 1 submits (not withholding,  $O_h, O_h, O_h, O_h$ ), the ISO calculates its expected procurement cost as  $\min\{H + S + A, 2H\} + \alpha \min\{H\} = (1 + \alpha)H + S + A$  and Supplier 1's expected payoff is  $\pi_1(\text{not withholding, } O_h, O_h, O_h, O_h) = (1 + \alpha)H$ . If Supplier 1 submits (withholding,  $O_l, O_l, O_h, O_h$ ), the ISO calculates its expected procurement cost as  $\min\{L + S + A\} + \alpha \min\{H\} = L + S + A + \alpha H$  and Supplier 1's expected payoff is  $\pi_1(\text{withholding, } O_l, O_l, O_h, O_h) = \alpha H + L$ . If Supplier 1 submits (withholding,  $O_h, O_l, O_h, O_h$ ), the ISO calculates its expected procurement cost as  $\min\{L + S + A\} + \alpha \min\{H\} = L + S + A + \alpha H$  and Supplier 1's expected payoff is  $\pi_1(\text{withholding, } O_h, O_l, O_h, O_h) = \alpha H + L$ . If Supplier 1 submits (withholding,  $O_h, O_h, O_h, O_h$ ), the ISO calculates its expected procurement cost as  $\min\{H + S + A\} + \alpha \min\{H\} = (1 + \alpha)H + S + A$  and Supplier 1's expected payoff is  $\pi_1(\text{withholding, } O_h, O_h, O_h, O_h) = (1 + \alpha)H$ . If Supplier 1 submits (withholding,  $O_l, O_h, O_h, O_h$ ), the ISO calculates its expected procurement cost as  $\min\{L + S + A\} + \alpha \min\{H\} = L + S + A + \alpha H$  and Supplier 1's expected payoff is  $\pi_1(\text{withholding, } O_l, O_h, O_h, O_h) = \alpha H + L$ . If Supplier 1 submits (withholding,  $O_h, O_h, O_h, O_h$ ), the ISO calculates its expected procurement cost as  $\min\{H + S + A\} + \alpha \min\{H\} = (1 + \alpha)H + S + A$  and Supplier 1's expected payoff is  $\pi_1(\text{withholding, } O_h, O_h, O_h, O_h) = (1 + \alpha)H$ . Therefore, (not withholding,  $O_l, O_h, O_h, O_h$ ), (not withholding,  $O_h, O_h, O_h, O_h$ ), (withholding,  $O_h, O_l, O_h, O_h$ ), and (withholding,  $O_h, O_h, O_h, O_h$ ) constitute equilibria; the day-ahead MCP is  $H$ , the real-time MCP is  $H$ , and the ISO's expected procurement cost is  $S + (2 + \alpha)H$  in equilibrium.

*Game 2: Payment Cost Minimization Formulation.*

If Supplier 1 submits (not withholding,  $O_l, O_l, O_h, O_h$ ), the ISO calculates its expected procurement cost as  $\min\{S + 2\max\{L, A\}, 2L\} + \alpha\min\{S + A\} = 2L + \alpha S + \alpha A$  and Supplier 1's expected payoff is  $\pi_1(\text{not withholding, } O_l, O_l, O_h, O_h) = 2L$ . If Supplier 1 submits (not withholding,  $O_h, O_l, O_h, O_h$ ), the ISO calculates its expected procurement cost as  $\min\{S + 2\max\{L, A\}, 2L\} + \alpha\min\{S + A\} = 2L + \alpha S + \alpha A$  and Supplier 1's expected payoff is  $\pi_1(\text{not withholding, } O_h, O_l, O_h, O_h) = 2L$ . If Supplier 1 submits (not withholding,  $O_l, O_h, O_h, O_h$ ), the ISO calculates its expected procurement cost as  $\min\{S + 2\max\{H, A\}, 2H\} + \alpha\min\{S + A\} = 2H + \alpha S + \alpha A$  and Supplier 1's expected payoff is  $\pi_1(\text{not withholding, } O_l, O_h, O_h, O_h) = 2H$ . If Supplier 1 submits (not withholding,  $O_h, O_h, O_h, O_h$ ), the ISO calculates its expected procurement cost as  $\min\{S + 2\max\{H, A\}, 2H\} + \alpha\min\{S + A\} = 2H + \alpha S + \alpha A$  and Supplier 1's expected payoff is  $\pi_1(\text{not withholding, } O_h, O_h, O_h, O_h) = 2H$ . If Supplier 1 submits (withholding,  $O_l, O_l, O_h, O_h$ ), the ISO calculates its expected procurement cost as  $\min\{S + 2\max\{L, A\}\} + \alpha\min\{H\} = 2L + S + \alpha H$  and Supplier 1's expected payoff is  $\pi_1(\text{withholding, } O_l, O_l, O_h, O_h) = \alpha H + L$ . If Supplier 1 submits (withholding,  $O_h, O_l, O_h, O_h$ ), the ISO calculates its expected procurement cost as  $\min\{S + 2\max\{H, A\}\} + \alpha\min\{H\} = (2 + \alpha)H + S$  and Supplier 1's expected payoff is  $\pi_1(\text{withholding, } O_h, O_l, O_h, O_h) = (1 + \alpha)H$ . If Supplier 1 submits (withholding,  $O_l, O_h, O_h, O_h$ ), the ISO calculates its expected procurement cost as  $\min\{S + 2\max\{L, A\}\} + \alpha\min\{H\} = 2L + S + \alpha H$  and Supplier 1's expected payoff is  $\pi_1(\text{withholding, } O_l, O_h, O_h, O_h) = \alpha H + L$ . If Supplier 1 submits (withholding,  $O_h, O_h, O_h, O_h$ ), the ISO calculates its expected procurement cost as  $\min\{S + 2\max\{H, A\}\} + \alpha\min\{H\} = (2 + \alpha)H + S$  and Supplier 1's expected payoff is  $\pi_1(\text{withholding, } O_h, O_h, O_h, O_h) = (1 + \alpha)H$ . Therefore, (not withholding,  $O_l, O_h, O_h, O_h$ ) and (not withholding,  $O_h, O_h, O_h, O_h$ ) constitute equilibria; the day-ahead MCP is  $H$ , the real-time MCP is  $A$ , and the ISO's expected procurement cost is  $2H + \alpha S + \alpha A$  in equilibrium.

In the equilibria found in Case 1, the ISO's expected procurement cost is lower under the payment cost minimization formulation,  $2H + \alpha S + \alpha A < S + (2 + \alpha)H$ .

**Case 2a.**  $L < S + A < H, 2L/(1 + \alpha) < H, A > L$ .

*Game 1: Offer Cost Minimization Formulation.*

Similar calculations to those performed under Case 1 show that the Supplier 1 strategies (not withholding,  $O_l, O_h, O_h, O_h$ ), (not withholding,  $O_h, O_h, O_h, O_h$ ), (withholding,  $O_h, O_l, O_h, O_h$ ), and (withholding,  $O_h, O_h, O_h, O_h$ ) constitute equilibria; Supplier 1's expected payoff is  $(1 + \alpha)H$ , the day-ahead MCP is  $H$ , the real-time MCP is  $H$ , and the ISO's expected procurement cost is  $(2 + \alpha)H + S$  in equilibrium.

*Game 2: Payment Cost Minimization Formulation.*

Similar calculations to those performed under Case 1 show that the Supplier 1 strategies (not withholding,  $O_l, O_h, O_h, O_h$ ) and (not withholding,  $O_h, O_h, O_h, O_h$ ) constitute equilibria; Supplier 1's expected payoff is  $2H$ , the day-ahead MCP is  $H$ , the real-time MCP is  $A$ , and the ISO's expected procurement cost is  $2H + \alpha S + \alpha A$  in equilibrium.

In the equilibria found in Case 2a, the ISO's expected procurement cost is lower under the payment cost minimization formulation,  $2H + \alpha S + \alpha A < (2 + \alpha)H + S$ .

**Case 2b.**  $L < S + A < H, 2L/(1 + \alpha) < H, A < L$ .

*Game 1: Offer Cost Minimization Formulation.*

Similar calculations to those performed under Case 1 show that the Supplier 1 strategies (not withholding,  $O_l, O_h, O_h, O_h$ ), (not withholding,  $O_h, O_h, O_h, O_h$ ), (withholding,  $O_h, O_l, O_h, O_h$ ), and (withholding,  $O_h, O_h, O_h, O_h$ ) constitute equilibria; Supplier 1's expected payoff is  $(1 + \alpha)H$ , the day-ahead MCP is  $H$ , the real-time MCP is  $H$ , and the ISO's expected procurement cost is  $(2 + \alpha)H + S$  in equilibrium.

*Game 2: Payment Cost Minimization Formulation.*

Similar calculations to those performed under Case 1 show that the Supplier 1 strategies (not withholding,  $O_l, O_h, O_h, O_h$ ) and (not withholding,  $O_h, O_h, O_h, O_h$ ) constitute equilibria; Supplier 1's expected payoff is  $2H$ ,



the day-ahead MCP is  $H$ , the real-time MCP is  $A$ , and the ISO's expected procurement cost is  $2H + \alpha S + \alpha A$  in equilibrium.

In the equilibria found in Case 2b, the ISO's expected procurement cost is lower under the payment cost minimization formulation,  $2H + \alpha S + \alpha A < (2 + \alpha)H + S$ .

**Case 3a.**  $L < S + A < H$ ,  $2L/(1 + \alpha) > H$ ,  $A > L$ .

*Game 1: Offer Cost Minimization Formulation.*

Similar calculations to those performed under Case 1 show that the Supplier 1 strategies (not withholding,  $O_l, O_l, O_h, O_h$ ) and (not withholding,  $O_h, O_l, O_h, O_h$ ) constitute equilibria; Supplier 1's expected payoff is  $2L$ , the day-ahead MCP is  $L$ , the real-time MCP is  $A$ , and the ISO's expected procurement cost is  $2L + \alpha S + \alpha A$  in equilibrium.

*Game 2: Payment Cost Minimization Formulation.*

Similar calculations to those performed under Case 1 show that the Supplier 1 strategies (not withholding,  $O_l, O_h, O_h, O_h$ ) and (not withholding,  $O_h, O_h, O_h, O_h$ ) constitute equilibria; Supplier 1's expected payoff is  $2H$ , the day-ahead MCP is  $H$ , the real-time MCP is  $A$ , and the ISO's expected procurement cost is  $2H + \alpha S + \alpha A$  in equilibrium.

In the equilibria found in Case 3a, the ISO's expected procurement cost is lower under the offer cost minimization formulation,  $2L + \alpha S + \alpha A < 2H + \alpha S + \alpha A$ .

**Case 3b.**  $L < S + A < H$ ,  $2L/(1 + \alpha) > H$ ,  $A < L$ .

*Game 1: Offer Cost Minimization Formulation.*

Similar calculations to those performed under Case 1 show that the Supplier 1 strategies (not withholding,  $O_l, O_l, O_h, O_h$ ) and (not withholding,  $O_h, O_l, O_h, O_h$ ) constitute equilibria; Supplier 1's expected payoff is  $2L$ , the day-ahead MCP is  $L$ , the real-time MCP is  $A$ , and the ISO's expected procurement cost is  $2L + \alpha S + \alpha A$  in equilibrium.

*Game 2: Payment Cost Minimization Formulation.*

Similar calculations to those performed under Case 1 show that the Supplier 1 strategies (not withholding,  $O_l, O_h, O_h, O_h$ ) and (not withholding,  $O_h, O_h, O_h, O_h$ ) constitute equilibria; Supplier 1's expected payoff is  $2H$ , the day-ahead MCP is  $H$ , the real-time MCP is  $A$ , and the ISO's expected procurement cost is  $2H + \alpha S + \alpha A$  in equilibrium.

In the equilibria found in Case 3b, the ISO's expected procurement cost is lower under the offer cost minimization formulation,  $2L + \alpha S + \alpha A < 2H + \alpha S + \alpha A$ .

**Case 4a.**  $H < A + S, A < L$ .

*Game 1: Offer Cost Minimization Formulation.*

Similar calculations to those performed under Case 1 show that the Supplier 1 strategies (not withholding,  $O_l, O_h, O_h, O_h$ ) and (not withholding,  $O_h, O_h, O_h, O_h$ ) constitute equilibria; Supplier 1's expected payoff is  $2H$ , the day-ahead MCP is  $H$ , the real-time MCP is  $A$ , and the ISO's expected procurement cost is  $2H + \alpha S + \alpha A$  in equilibrium.

*Game 2: Payment Cost Minimization Formulation.*

Similar calculations to those performed under Case 1 show that the Supplier 1 strategies (not withholding,  $O_l, O_h, O_h, O_h$ ) and (not withholding,  $O_h, O_h, O_h, O_h$ ) constitute equilibria; Supplier 1's expected payoff is  $2H$ , the day-ahead MCP is  $H$ , the real-time MCP is  $A$ , and the ISO's expected procurement cost is  $2H + \alpha S + \alpha A$  in equilibrium.

In the equilibria found in Case 4a, the ISO's expected procurement cost is the same under both formulations.

**Case 4b.**  $H < A + S, L < A < H$ .

*Game 1: Offer Cost Minimization Formulation.*

Similar calculations to those performed under Case 1 show that the Supplier 1 strategies (not withholding,  $O_l, O_h, O_h, O_h$ ) and (not withholding,  $O_h, O_h, O_h, O_h$ ) constitute equilibria; Supplier 1's expected payoff is  $2H$ , the day-ahead MCP is  $H$ , the real-time MCP is  $A$ , and the ISO's expected procurement cost is  $2H + \alpha S + \alpha A$  in equilibrium.

*Game 2: Payment Cost Minimization Formulation.*

Similar calculations to those performed under Case 1 show that the Supplier 1 strategies (not withholding,  $O_l, O_h, O_h, O_h$ ) and (not withholding,  $O_h, O_h, O_h, O_h$ ) constitute equilibria; Supplier 1's expected payoff is  $2H$ , the day-ahead MCP is  $H$ , the real-time MCP is  $A$ , and the ISO's expected procurement cost is  $2H + \alpha S + \alpha A$  in equilibrium.

In the equilibria found in Case 4b, the ISO's expected procurement cost is the same under both formulations.

**Case 4c.**  $H < A + S, H < A < (2 - \alpha)H$ .

*Game 1: Offer Cost Minimization Formulation.*

Similar calculations to those performed under Case 1 show that the Supplier 1 strategies (not withholding,  $O_l, O_h, O_h, O_h$ ) and (not withholding,  $O_h, O_h, O_h, O_h$ ) constitute equilibria; Supplier 1's expected payoff is  $2H$ , the day-ahead MCP is  $H$ , the real-time MCP is  $A$ , and the ISO's expected procurement cost is  $2H + \alpha S + \alpha A$  in equilibrium.

*Game 2: Payment Cost Minimization Formulation.*

Similar calculations to those performed under Case 1 show that the Supplier 1 strategies (not withholding,  $O_l, O_h, O_h, O_h$ ) and (not withholding,  $O_h, O_h, O_h, O_h$ ) constitute equilibria; Supplier 1's expected payoff is  $2H$ , the day-ahead MCP is  $H$ , the real-time MCP is  $A$ , and the ISO's expected procurement cost is  $2H + \alpha S + \alpha A$  in equilibrium.

In the equilibria found in Case 4c, the ISO's expected procurement cost is the same under both formulations.

**Case 4d.**  $H < A + S$ ,  $A > (2 - \alpha)H$ .

*Game 1: Offer Cost Minimization Formulation.*

Similar calculations to those performed under Case 1 show that the Supplier 1 strategies (withholding,  $O_l, O_l, O_h, O_h$ ), (withholding,  $O_h, O_l, O_h, O_h$ ), (withholding,  $O_l, O_h, O_h, O_h$ ), and (withholding,  $O_h, O_h, O_h, O_h$ ) constitute equilibria; Supplier 1's expected payoff is  $A + \alpha H$ , the day-ahead MCP is  $A$ , the real-time MCP is  $H$ , and the ISO's expected procurement cost is  $S + 2A + \alpha H$  in equilibrium.

*Game 2: Payment Cost Minimization Formulation.*

Similar calculations to those performed under Case 1 show that the Supplier 1 strategies (withholding,  $O_l, O_l, O_h, O_h$ ), (withholding,  $O_h, O_l, O_h, O_h$ ), (withholding,  $O_l, O_h, O_h, O_h$ ), and (withholding,  $O_h, O_h, O_h, O_h$ ) constitute equilibria; Supplier 1's expected payoff is  $A + \alpha H$ , the day-ahead MCP is  $A$ , the real-time MCP is  $H$ , and the ISO's expected procurement cost is  $S + 2A + \alpha H$  in equilibrium.

In the equilibria found in Case 4d, the ISO's expected procurement cost is the same under both formulations.

In Games 1 and 2 above, we see that, in equilibrium, payment cost minimization delivers relatively lower procurement costs to the ISO than does offer cost minimization in two of the four cases (Cases 1 and 2). In one case, payment cost minimization delivers relatively higher procurement costs to the ISO than does offer cost minimization in equilibrium (Case 3). Finally, in one case, both formulations deliver the same procurement costs to the ISO in equilibrium (Case 4). In summary, on the basis of energy procurement costs in equilibrium, payment cost minimization outperforms offer cost minimization two-to-one.<sup>4</sup>

Figure 3 summarizes these results graphically in  $(L, H)$  space. The figure illustrates the regions of Supplier 1's offer price strategy space that correspond to the four cases analyzed throughout this section. Note first that the numerical example encountered in Example Games 1 and 2 of Section 2 corresponds to Case 3, where, counter to intuition, the offer cost minimiza-

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<sup>4</sup>The less likely knife-edge cases, such as  $S + A = L$ , are omitted from our analysis.

tion formulation outperforms the payment cost minimization formulation. Second, the sizes of the various regions in Figure 3 primarily depend upon the relationship between Supplier 1's offer prices ( $L$  and  $H$ ) and Supplier 2's start-up costs and offer price ( $S$  and  $A$ , respectively). However, the region corresponding to Case 3 also depends upon  $\alpha$ , the probability of a unit demand for wholesale electricity in the real-time market. Two extreme values of  $\alpha$  are worth considering more closely. When  $\alpha = 0$ , there is no real-time market, and thus the model collapses to a single day-ahead market, reproducing the results of Knoblauch (2005, Section 3). When  $\alpha = 1$ , there is a real-time market with certainty, and examination of Figure 3 reveals that Case 3 disappears; that is, when  $\alpha = 1$ , there are *no* cases where the offer cost minimization formulation outperforms the payment cost minimization formulation on the basis of procurement costs.

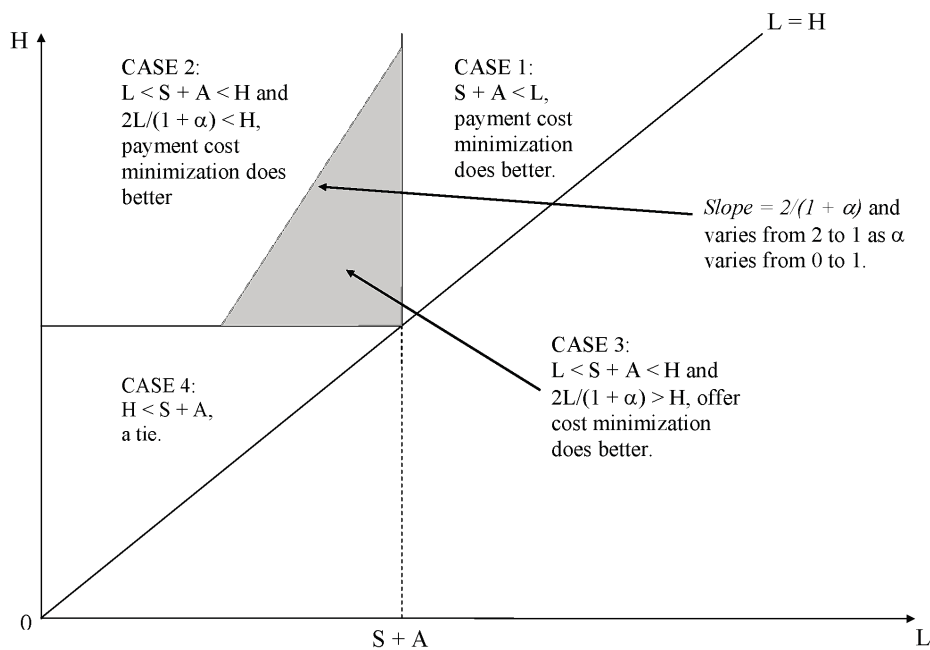


Figure 3: Procurement Costs in Two-Supplier Auctions

## 3.2 Efficiency

We can carry the analysis of Section 3.1 further and examine the efficiency implications of Supplier 1's strategic withholding and offer pricing behavior under the offer cost and payment cost minimization formulations. We evaluate productive efficiency as is standard in the economics literature on auctions. Efficiency conditions hold if there is no lower cost generation out of operation while a higher cost generator is in operation; this is the preservation of the so-called *merit order* based upon generator production costs. On the other hand, efficiency conditions are violated if there is a lower cost generator out of operation while a higher cost generator is in operation.

We examine the procurement decisions reached in equilibrium under both the offer cost and payment cost minimization formulations for each of the four cases and rank the formulations accordingly based upon whether or not they violate the aforementioned efficiency conditions. We evaluate efficiency performance for the day-ahead market only, as the structure of the model renders such an evaluation for the real-time market trivial. Moreover, when only *withholding* equilibria obtain for a particular case (e.g., Case 4d), the efficiency conditions hold trivially. And, when a particular case has both *withholding* and *non-withholding* equilibria, we obtain efficiency results for the *non-withholding* equilibria (e.g., Case 1, Game 1: Offer Cost Minimization Formulation).

Our analysis of efficiency largely follows the setup of Section 3.1 and the same four cases discussed there. Here, however, we suppose that Supplier 1's generation costs are  $L > 0$  and that Supplier 2's generation costs are  $S + A > 0$ . The four cases naturally suggest the order relationships between the suppliers' costs, and we need only examine how the two minimization formulations award contracts in the day-ahead market and whether or not they violate merit order.

Under the conditions of Case 1, Supplier 1 has relatively higher generation costs than does Supplier 2,  $L > S + A$ , and so the ISO should not select both energy units from Supplier 1 in the day-ahead market if its allocation is to be efficient. Offer cost minimization selects one unit from Supplier 1 and one unit from Supplier 2 in the day-ahead market and is thus efficient. Payment cost minimization, however, selects both units from Supplier 1 in the day-ahead market and is thus inefficient.

In Case 2, Supplier 1 has relatively lower generation costs than does Supplier 2,  $L < S + A$ . So, the ISO should select both energy units from

Supplier 1 in the day-ahead market if its allocation is to be efficient. Offer cost minimization selects one unit from Supplier 1 and one unit from Supplier 2 in the day-ahead market and is thus inefficient. Payment cost minimization, however, selects both units from Supplier 1 in the day-ahead market and is thus efficient.

In Case 3, Supplier 1 has relatively lower generation costs than does Supplier 2,  $L < S + A$ . So, the ISO should select both energy units from Supplier 1 in the day-ahead market if its allocation is to be efficient. Both offer cost and payment cost minimization select both units from Supplier 1 in the day-ahead market and are thus efficient.

Lastly, in Case 4, Supplier 1 has relatively lower generation costs than does Supplier 2,  $L < S + A$ . So, the ISO should select both energy units from Supplier 1 in the day-ahead market if its allocation is to be efficient. Both offer cost and payment cost minimization select both units from Supplier 1 in the day-ahead market and are thus efficient.<sup>5</sup>

In Games 1 and 2 above, we see that, in equilibrium, payment cost minimization produces an efficient allocation of energy contracts in the day-ahead market in three of the four cases (Cases 2, 3, and 4). Offer cost minimization also achieves efficiency in three cases (Cases 1, 2, and 4). In one case, payment cost minimization delivers an inefficient allocation while offer cost minimization delivers an efficient allocation (Case 1); and, in one case, we have the *vice versa* (Case 3). In two cases, both formulations achieve efficient allocations (Cases 3 and 4). Finally, there are no cases where both formulations achieve inefficient allocations in the day-ahead market. Figure 4 summarizes these results graphically in  $(L, H)$  space.

### 3.3 Discussion

Figures 3 and 4 provide evidence of a tradeoff that is by now well-known in the economics literature on auctions. Ausubel and Cramton (2002) prove that multi-unit discriminatory and uniform price auctions cannot be unambiguously ranked on the basis of both revenues and efficiency.<sup>6</sup> We obtain

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<sup>5</sup>As mentioned above, Case 4d is an exception because all of its equilibria feature withholding on the part of Supplier 1, which makes the evaluation of efficiency in the day-ahead market trivial.

<sup>6</sup>The revenue criterion in the sale auctions Ausubel and Cramton (2002) analyze is the analogue of the procurement cost criterion we consider above in the context of procurement auctions for wholesale electricity. Fabra et al. (2004) obtain analogous results in

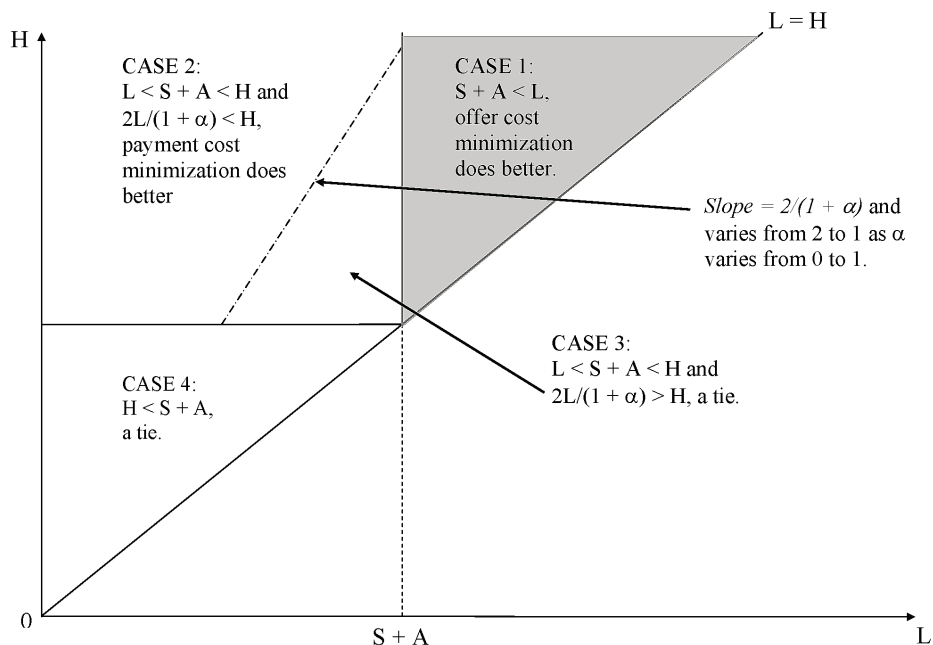


Figure 4: Efficiency in Two-Supplier Auctions

a similar result for the offer cost and payment cost minimization formulations operating in the simple two-supplier auctions for wholesale electricity analyzed throughout this section.

Comparing the procurement cost and efficiency outcomes for Case 1 in Figures 3 and 4, respectively, reveals that we *cannot* unambiguously rank the offer cost minimization and payment cost minimization formulations on the basis of both procurement costs and efficiency. Specifically, in Case 1, payment cost minimization delivers lower expected procurement costs to the ISO than does offer cost minimization, but in doing so provides a dispatch that is inefficient while that selected by offer cost minimization is efficient. In the next section, we will see that this procurement cost-efficiency tradeoff persists when we proxy for the effects of increased competition by adding a Bertrand competitor to our simple auctions for wholesale electricity.

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the context of a model of wholesale electricity auction markets with discriminatory and uniform pricing.



## 4 An Added Competitor: Three-Supplier Wholesale Electricity Auctions

### 4.1 Procurement Costs

A good way to proxy for the effects of increased competition in small markets is to introduce a Bertrand competitor. We consider a wholesale electricity auction with three suppliers and one ISO. Again, as in Section 3, this market features a dual settlement system; that is, this wholesale electricity market features both a day-ahead and real-time market for electricity.

Suppliers 1 and 2 are our Bertrand competitors and can each supply 0, 1, or 2 units of energy, have start-up costs of zero, unit costs of  $L > 0$ , and select offer prices  $O_i(1) = O_i(2) = p_i \in \mathfrak{R}_+$ ,  $i = 1, 2$ . Further, Suppliers 1 and 2 may choose whether or not to withhold one unit of energy capacity from the day-ahead market for exclusive use in the real-time market. Supplier 3, on the other hand, can supply 0 or 1 units of energy, has start-up costs of  $S > 0$  dollars, and has one offer price,  $O_3(1) = A > 0$  dollars per unit. Supplier 3 always offers its unit of capacity for sale in the day-ahead market.

Fix some number  $N > L$ . In the day-ahead market, the ISO has a demand for 2 units of electricity so long as the MCP for 2 units is less than  $N$ , 1 unit if the MCP for 2 units is greater than or equal to  $N$  and the MCP for 1 unit is less than  $N$ , and 0 units otherwise. In the real-time market, with probability  $\alpha \in (0, 1)$ , the ISO has a demand for 1 unit of electricity so long as the MCP for 1 unit is less than  $N$  and 0 units otherwise, and with probability  $1 - \alpha$ , the ISO has a demand for 0 units. For mathematical convenience, any energy offers that are not selected to run in the day-ahead market “roll over” to the real-time market without adjustment to their offer prices.

Unfortunately, the addition of capacity withholding strategies to Supplier 1 and 2’s offer pricing strategies results in an auction pricing game that is much more complex than the standard Bertrand game. In particular, the payoff functions for Suppliers 1 and 2 do not have expressions conducive to stating and proving formal propositions about equilibrium pricing and withholding behavior. However, on the strength of the standard Bertrand competition result and the more recent result of Knoblauch (2005, Proposition 1), we proceed to simulate the effects of increased competition in wholesale electricity markets by assuming that our Bertrand competitors are each driven to

price at their unit costs,  $L$ .<sup>7</sup> The solutions to Games 3 and 4 break naturally into two cases.

**Case I.**  $S + A < L$ .

*Game 3: Offer Cost Minimization Formulation.*

If Suppliers 1 and 2 are pricing at their unit costs, they are each indifferent between not withholding and withholding a unit of energy capacity from the day-ahead market. We suppose that they each choose not to withhold capacity. In this case, the ISO calculates its expected procurement cost as  $\min\{S + A + L, 2L\} + \alpha \min\{L\} = S + A + (1 + \alpha)L$  by selecting 1/2 a unit from each of Supplier 1 and 2 and 1 unit from Supplier 3 in the day-ahead market and 1/2 a unit from each of Supplier 1 and 2 in the real-time market. The day-ahead MCP is  $L$ , the real-time MCP is  $L$ , and the ISO's expected procurement cost is  $(2 + \alpha)L + S$ .

*Game 4: Payment Cost Minimization Formulation.*

In this case, the ISO calculates its expected procurement cost as  $\min\{S + 2\max\{L, A\}, 2L\} + \alpha \min\{S + A, L\} = 2L + \alpha S + \alpha A$  by selecting 1 unit from each of Supplier 1 and 2 in the day-ahead market and 1 unit from Supplier 3 the real-time market. The day-ahead MCP is  $L$ , the real-time MCP is  $A$ , and the ISO's expected procurement cost is  $2L + \alpha S + \alpha A$ .

In Case I, the ISO's expected procurement cost is lower under the payment cost minimization formulation,  $2L + \alpha S + \alpha A < (2 + \alpha)L + S$ .

**Case II.**  $L < S + A$ .

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<sup>7</sup>In the context of a similar single-market wholesale electricity model, Knoblauch (2005, Proposition 1) finds that  $(L, L)$  emerges as the *unique Nash equilibrium* among the Bertrand competitors. The result is striking in that her Bertrand competition games do not contain *any* mixed strategy Nash equilibria. The two-market auction model analyzed throughout this paper is a direct extension of Knoblauch (2005) and it thus seems reasonable to suppose that the pure strategy Bertrand competition outcome is likely to emerge as an equilibrium outcome, possibly one among many, of our three-supplier auction games analyzed in this section.

*Game 3: Offer Cost Minimization Formulation.*

In this case, the ISO calculates its expected procurement cost as  $\min\{S + A + L, 2L\} + \alpha \min\{S + A, L\} = (2 + \alpha)L$  by selecting 1 unit from each of Supplier 1 and 2 in the day-ahead market and 1/2 a unit from each of Supplier 1 and 2 in the real-time market. The day-ahead MCP is  $L$ , the real-time MCP is  $L$ , and the ISO's expected procurement cost is  $(2 + \alpha)L$ .

*Game 4: Payment Cost Minimization Formulation.*

In this case, the ISO calculates its expected procurement cost as  $\min\{S + 2\max\{L, A\}, 2L\} + \alpha \min\{S + A, L\} = (2 + \alpha)L$  by selecting 1 unit from each of Supplier 1 and 2 in the day-ahead market and 1/2 a unit from each of Supplier 1 and 2 in the real-time market. The day-ahead MCP is  $L$ , the real-time MCP is  $L$ , and the ISO's expected procurement cost is  $(2 + \alpha)L$ .

In Case II, the ISO's expected procurement cost is the same under both formulations.

In Games 3 and 4 above, we see that payment cost minimization delivers relatively lower procurement costs to the ISO than does offer cost minimization in one of the two cases (Case I), and, in one case, both formulations deliver the same procurement costs to the ISO (Case II). Thus, under conditions proxying for increased competition, Bertrand competition in our simple three-supplier wholesale electricity auctions, payment cost minimization *never* does worse than offer cost minimization and sometimes outperforms it on the basis of energy procurement costs. Figure 5 summarizes these results graphically in  $(L, H)$  space.

## 4.2 Efficiency

We can carry the analysis of Section 4.1 further and examine the efficiency implications of adding a Bertrand competitor to the basic two-supplier model of Section 3 and simulating the effects of increased competition in wholesale electricity auction markets. Again, we evaluate productive efficiency as above in Section 3.1. Efficiency conditions hold if there is no lower cost generation out of operation while a higher cost generator is in operation; this is the preservation of the so-called *merit order* based upon generator production costs. On the other hand, efficiency conditions are violated if there is a lower

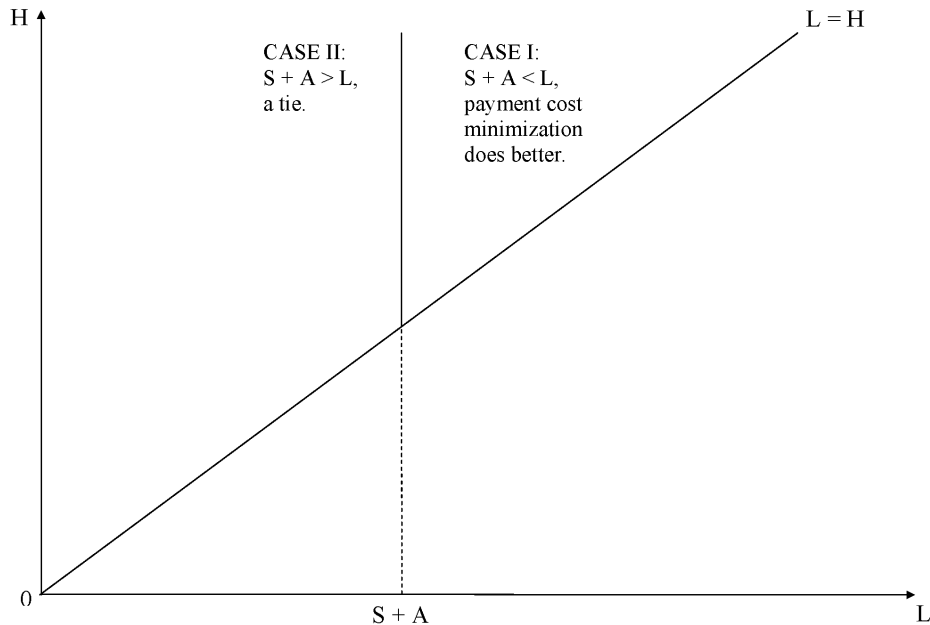


Figure 5: Procurement Costs in Three-Supplier Auctions

cost generator out of operation while a higher cost generator is in operation.

We examine the procurement decisions reached under both the offer cost and payment cost minimization formulations when both Suppliers 1 and 2 are pricing at their marginal costs of  $L > 0$  and are thus indifferent between withholding and not withholding. We analyze each of the two cases from Section 4.1 and rank the formulations accordingly based upon whether or not they violate the aforementioned efficiency conditions.

Our analysis of efficiency largely follows the setup of Section 4.1 and the same two cases discussed there. We suppose that Supplier 1 and Supplier 2 have generation costs of  $L > 0$  and that Supplier 3's generation costs are  $S + A > 0$ . The two cases naturally suggest the order relationships between the suppliers' costs, and we need only examine how the two minimization formulations award contracts in the day-ahead market and whether or not they violate merit order.

Under the conditions of Case I, Suppliers 1 and 2 have relatively higher

generation costs than does Supplier 3,  $L > S + A$ , and so the ISO should not select both energy units from Suppliers 1 and 2 in the day-ahead market if its allocation is to be efficient. Offer cost minimization selects 1/2 a unit from each of Supplier 1 and 2 and 1 unit from Supplier 3 in the day-ahead market and is thus efficient. Payment cost minimization, however, selects 1 unit from each of Supplier 1 and 2 in the day-ahead market and is thus inefficient.

In Case II, Suppliers 1 and 2 have relatively lower generation costs than does Supplier 3,  $L < S + A$ . So, the ISO should select both energy units from Suppliers 1 and 2 in the day-ahead market if its allocation is to be efficient. Offer cost minimization selects 1 unit from each of Supplier 1 and 2 in the day-ahead market and is thus efficient. Payment cost minimization also selects 1 unit from each of Supplier 1 and 2 in the day-ahead market and is thus efficient.

In Games 3 and 4 above, we see that when simulating the effects of increased competition, payment cost minimization produces an efficient allocation of energy contracts in the day-ahead market in one of the two cases (Case II). Offer cost minimization achieves efficiency in two cases (Cases I and II). In one case, payment cost minimization delivers an inefficient allocation while offer cost minimization delivers an efficient allocation (Case I); and, in one case, we have a tie (Case II). Finally, there are no cases where both formulations achieve inefficient allocations in the day-ahead market. Figure 6 summarizes these results graphically in  $(L, H)$  space.

### 4.3 Discussion

Figures 5 and 6 provide further evidence of the procurement cost-efficiency tradeoff that emerged above in the context of our two-supplier auction model of Section 3. Again, we *cannot* unambiguously rank the offer cost and payment cost minimization formulations on the basis of both procurement costs and efficiency in our three-supplier auctions with Bertrand competitors proxying for the effects of increased competition.

Comparing the procurement cost and efficiency outcomes for Case I in Figures 5 and 6, respectively, reveals that we cannot unambiguously rank the offer cost minimization and payment cost minimization formulations on the basis of both procurement costs and efficiency. Specifically, in Case I, payment cost minimization delivers lower expected procurement costs to the ISO than does offer cost minimization, but in doing so provides a dispatch

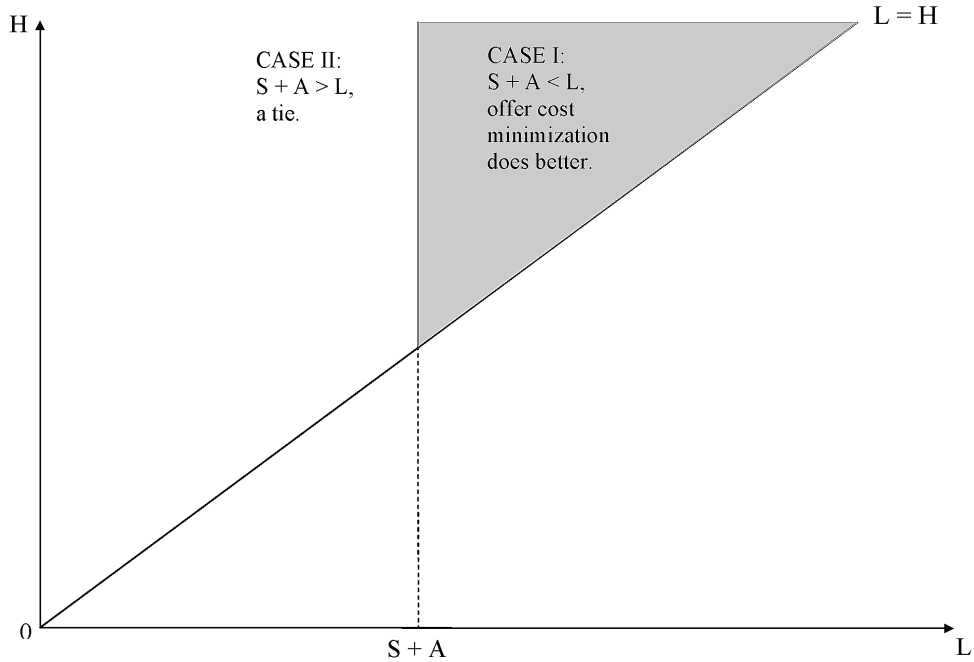


Figure 6: Efficiency in Three-Supplier Auctions

that is inefficient while that selected by offer cost minimization is efficient. This case is defined by  $S + A < L$ , the same relationship that defines Case 1 from the two-supplier auction model of Section 3 where we first identified this procurement cost-efficiency tradeoff. Thus, it is clear that this tradeoff persists even when we expand the model to simulate the effects of increased competition in auction markets for wholesale electricity.

## 5 Conclusion

In this paper, we examine the procurement cost-minimizing and productive efficiency performance of offer cost minimization, the current practice among ISOs in wholesale electricity markets in the U.S., and payment cost minimization, a proposed alternative. In the context of simple, two-supplier auction markets, we find that, in a majority of cases, payment cost minimization

delivers relatively lower procurement costs to the ISO than does offer cost minimization even when accounting for generators' strategic offer pricing and capacity withholding behavior. There is, however, one case where payment cost minimization delivers relatively *higher* procurement costs to the ISO (Case 3). We simulate the effects of increased competition in wholesale electricity markets by introducing Bertrand competition between two symmetric suppliers in the context of simple three-supplier auctions and find that payment cost minimization either ties or dominates offer cost minimization in terms of the procurement costs it returns to the ISO in equilibrium.

Our analysis of the productive efficiency of offer cost minimization and payment cost minimization in two-supplier auction markets reveals a trade-off between the goals of procurement cost minimization and productive efficiency, as the case where offer cost minimization achieves greater efficiency than payment cost minimization (Case 1) is a case where payment cost minimization achieves lower procurement costs in equilibrium. Thus, we *cannot* unambiguously rank the two allocation mechanisms in terms of both procurement costs and productive efficiency. More importantly, this tradeoff persists when we proxy for the effects of increased competition in these auction markets by introducing Bertrand competition.

There are a number of areas for future research in this vein. Controlled experiments examining the impact on both procurement costs and productive efficiency of switching between offer cost minimization and payment cost minimization could provide valuable insight into how participants in these markets tailor their behavior to changes in the auction format as well as provide empirical tests of the results we develop in this paper. By getting away from the demands of classical equilibrium analysis, numerical simulations and studies featuring agent-based modeling in the spirit of Nicolaisen et al. (2001) and Kian et al. (2005) can examine the issues at the heart of this paper in the context of wholesale electricity auction markets of greater complexity and realism with greater hope of remaining tractable. Lastly, since demand response is a great concern among economists interested in wholesale electricity markets (Borenstein (2002) and Hunt (2002)), and since many of these auction markets are in actuality double auctions, future research could easily extend the models of Knoblauch (2005) as well as those that we develop in this paper to the context of double auctions and examine the performance of offer cost and payment cost minimization on the basis of procurement costs, productive efficiency, consumption efficiency, and social welfare.

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