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Abstract

This paper develops a reduced form three-factor model which includes a liquidity proxy of market conditions which is then used to provide implicit prices. The model prices are then compared with observed market prices of credit default swaps to determine if swap rates adequately reflect market risks. The findings of the analysis illustrate the importance of liquidity in the valuation process. Moreover, market liquidity, a measure of investors' willingness to commit resources in the credit default swap (CDS) market, was also found to improve the valuation of investors' autonomous credit risk. Thus a failure to include a liquidity proxy could underestimate the implied autonomous credit risk. Autonomous credit risk is defined as the fractional credit risk which does not vary with changes in market risk and liquidity conditions.

Keywords: Credit Default Swaps; Market Liquidity; Bid-Ask Spreads; Autonomous Credit Risk, Risk Premium

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US Corporate Default Swap Valuation: The Market liquidity hypothesis and Autonomous Credit Risk

1. Introduction to CDS Valuation

Market liquidity has become central to debt markets over the last ten years, as evidenced by the number of innovative credit products that have been introduced to increase the depth and breadth of credit markets. This phenomenon has led to the exponential growth of credit derivatives, one of the most successful financial innovative products of the past decade. The British Bankers Association estimates that the credit derivative market grew from a notional \$180 billion in 1997 to \$5.0 trillion in 2004 and is expected to reach upwards of \$8.2 trillion in 2006². A review of the credit markets has shown that while overall quality of global credit has deteriorated the volume of corporate bonds (corporate credit risk) has risen dramatically over the past few years.

The growing importance of the corporate debt component of the overall global debt market (relative to government debt) indicates a growth in global credit risk, which in itself partly explains the observed exponential growth in the use of credit default swaps to mitigate this growing counterparty risk. Much of this growth in counter-party risk stems from the ever increasing demand by banks, insurance companies, institutional investors and hedge funds seeking credit risk insurance to cover risky long bond exposures. As far back as November 2002, then Fed Chief Alan Greenspan appearing before the Foreign Relations Committee suggested that

² In an August 31st 2006 Wall Street Journal article "Can Anyone Police the Swaps" the current CDS market was estimated at upwards of \$17 trillion.

one positive outcome of this growth is the strengthening of the financial sector by spreading credit risk more broadly across the entire sector as against having it all concentrated among a few participants.

The abundance of current financial literature on credit default swap³ valuation ignores the possible role of market liquidity as a parameter in valuing credit default swaps (credit risks). Empirical work by Fleming (2003) shows that bond market liquidity can be a barometer of market conditions, which can signal the willingness of market makers to commit capital and take on risk. While there is an abundance of one or two-factor models that attempt to value these instruments, there is a lack of work that factors the impact of market liquidity in the valuation model. Chen *et al* (2004) found that changes in liquidity and credit ratings alone explain 33% of cross-sectional variations in investment and speculative grade bond spreads. Recent work by Tang *et al* (2006) and Acharya *et al* (2005) has also concluded that there is evidence that liquidity plays a critical role in the CDS market. Given the importance of liquidity in bond pricing, it is believed that the introduction of this parameter in the credit default swap (CDS) pricing model will help to improve the explanatory power of the model and better explain the cross-sectional variation in the CDS spreads.

³ A credit default swap (CDS) may be described as an insurance contract that provides counter party protection in the event of a default by an underlying referenced issuer. See Das and Hanouna (2006) for a discussion of CDS spreads and products.

1.1 Approaches to Credit Default Swap Valuation

Earlier empirical work on valuing credit default swaps (CDS) and other contingent claims that are subject to default risks have been modeled using either the structural or the reduced model approach. The structural form model views contingent claims as options written on the value of an underlying firm's assets. In so doing the model treats the bankruptcy or default process as endogenous by explicitly modeling the asset and liability structure of the company. Stochastic processes for both the value of assets and liabilities are specified and default is triggered whenever the value of assets falls below the value of liabilities. In other words, default endogenously occurs when the debt value of the firm exceeds the total value of the firm. Duffie and Singleton (1999) also suggest that these models are based on first passage of assets to a default boundary. These models have been used by Merton (1974), Chance (1990), Longstaff and Schwartz (1995), Bharath and Shumway (2004), Zhang, Zou and Zhu (2005) and Das, Hanouna and Sarin (2006).

Adaptations of structural models are based on the original framework developed by Merton (1974) using the principles of option pricing. In such a framework, the default process of a company is driven by the value of the company's assets and the risk of a firm's default is therefore explicitly linked to the variability in the firm's asset value. The Merton model premises that default occurs when the value of a firm's assets is lower than that of its liabilities. Assuming that the

company's debt is entirely represented by a zero-coupon bond, if the value of the firm at maturity is greater than the face value of the bond, then the bondholder gets back the face value of the bond. However, if the value of the firm is less than the face value of the bond, the equity holders get nothing and the bondholders get back the market value of the firm. The payoff at maturity of the bondholder is therefore equivalent to the face value of the bond minus a put option on the value of the firm, with a strike price equal to the face value of the bond and a maturity equal to the maturity of the bond. Following this basic intuition, Merton derived an explicit formula for default risky bonds, which can be used both to estimate the probability of default (PD) of a firm and to estimate the yield differential between a risky bond and a default-free bond⁴.

However, despite improvements over the years to make the original Merton framework more robust, the improved structural-form model still suffers from three main drawbacks, which represent the main reasons behind their relatively poor empirical performance. They still require estimates for the parameters of the firm's asset value, which is non-observable. Also, they cannot incorporate credit rating changes that occur quite frequently for default risky corporate debts. Finally, most structural form models assume that the value of the firm is continuous in time. As a result, the time of default can be predicted just before it happens and thus no surprise events occur.

⁴ In addition to Merton (1974), first generation structural-form models include Black and Cox (1976), Geske (1977), and Vasicek (1984). Each of these models tries to refine the original Merton Framework by removing one or more of the unrealistic assumptions. Black and Cox (1976) introduces the possibilities of more complex capital structures, with subordinate debt; Geske (1977) introduces interest paying debt; Vasicek (1984) introduces the distinction between short and long term liabilities, which now represents a distinctive feature of Kamakura Corp's KMV model.

In attempting to overcome the difficulties of implementing and working with structural models, researchers introduced reduced form models that use simple valuation procedures to produce exceptional results, that compare well to their structural counterparts. Reduced form models achieve this feat because they assume that default is an unpredictable event following some exogenous unexpected random jump process and as such does not require estimates of the value of the firm's assets (see Longstaff *et al* (2005)). Though recent work by Arora et al (2005) suggests that reduced form models do not significantly outperform structural models when predicting probability of default and CDS premia, the reduced form model's ease of calibration, simplicity in specification and limited data requirement makes them popular. These models include those developed by Jarrow and Turnbull (1995), Jarrow, Lando and Turnbull (1997), Duffie, Saita and Wang (2004), Longstaff, Mithal and Neis (2005) and Wu and Zang (2005).

Reduced form models have been extended in different directions over the years so as to improve their effectiveness in predicting default and valuing credit risk. However, a review of the literature has not revealed an attempt to model the effect of market liquidity in the valuation of credit risk. Longstaff, Mithal and Neis (2005) used the bond market to extract data which was used to price CDS contracts. They found that bond implied CDS premia were higher than market CDS spreads. The difference was attributed to liquidity factors and tax effects which do not necessarily reflect the default risk of the underlying asset. Blanco, Brennan, and Marsh (2005) reported that though they found that implied CDS and bond premia

appeared in line with each other in the long run, CDS spreads tend to respond more quickly to changes in credit conditions in the short run. This short run phenomenon appears to be a market liquidity condition. In addition, Tang and Yan (2006) found from their work on the effects of liquidity in the CDS market that CDS spreads were significantly positive with liquidity, which led them to conclude that there is significant liquidity effects in the CDS market.

The purpose of this study is to extend the popular reduced form approach to modeling credit risk by including a market liquidity proxy. Observed CDS market data tend to suggest that liquidity is directly related to credit quality. So while the study will be looking at the direct effects of liquidity in the valuation process, it will also be interesting to view the indirect effects of liquidity on autonomous credit risk. The proposed methodology for this study builds on the recent literature of credit default swap valuation through joint default risk-neutral intensity and recovery rate dynamics. The study adopts historical CDS bid-ask spreads as a proxy of market liquidity.

The study's main innovations are, (a) the effects of liquidity in CDS valuation, examining how changing liquidity conditions affect credit risk valuation in high frequency data; and (b) evaluating the effects of liquidity on the phenomenon of autonomous credit risk. Autonomous credit risk appears to be a positive phenomenon that occurs when market risk (short term instantaneous interest rate) is zero and markets are fully liquid. Such credit risk is considered autonomous of market risk and market liquidity only when the credit spreads or premia does not vary with

changes in market risk and liquidity conditions. An absence of liquidity effects in the market should result in an underestimation of the autonomous credit risk. In a somewhat similar study Pan and Singleton (2005) found that in the absence of liquidity considerations in the CDS market, implied recovery rates were significantly lower than generally observed.

These results should thus compliment past empirical work in better estimating credit risk. The study will begin by laying out the proposed model structure and components of the model following the two-factor model developed by Jarrow *et al* (2001). In the initial specification of the model the hazard rate is only a function of the default-free interest rate. The extended three-factor model⁵ will include a market liquidity proxy. Liquidity spreads are increasingly being viewed as a function of the volatility of the firm's assets and leverage, which are key determinants of credit risk. Its effects tend to be most dramatic as it tends to disappear when most needed by the market (as demonstrated in some of the more recent market downturns) and suddenly re-emerges as markets become bullish. Both specifications will be used in the pricing of market traded US corporate credit default swap. The estimates of the parameters will be compared across the standard and extended forms of the reduced-form two-factor pricing model, across the observed dataset, and to the estimates obtained in similar studies to determine the relevance of these assumptions to the outcome of the results.

Economic theory suggests that market and credit risks are intrinsically related to each other and inseparable. If the market value of the firm's assets unexpectedly

⁵ As discussed in Section 2, the two-factor model is a special case of the three-factor liquidity enhanced model.

changes, generating market risk, this will increase the probability of default thereby generating credit risk⁶. As such, a central hypothesis to a number of reduced form models and the one used in this study is the fact that market and credit risks are positively related, inseparable and dependent on the macro economy (short term interest rates). However these models appear to be missing another important aspect; market liquidity considerations, which can be viewed as a function of the volatility of the firm's assets and leverage, a determinant of credit risk that measures the willingness of market makers to commit resources to absorb market volume.

The remainder of the paper is organized into four sections. Section 2 introduces the theoretical foundations of the model and discusses the methodology for credit default swap pricing, giving some overview of current valuation methodologies and the analytical procedure for including the liquidity proxy to the pricing process. Section 3 gives a brief description of the CDS data and the various explanatory variables. Section 4 presents the main empirical findings regarding the role of market liquidity in CDS valuation. Section 5 summarizes the finding and proposes areas of future research.

⁶ Credit risk is jointly determined by the occurrence of default and the recovered amounts in the event of a default.

2. Model Structure

This section lays out the model structure and the component parts of the model following the two-factor reduced form approach developed by Jarrow *et al* (2001). The two-factor model is a special case of the three-factor liquidity enhanced specification. When $l_i = 0$ the three-factor model collapses to the Jarrow *et al*'s (2001) two-factor framework. The model assumes that markets are frictionless, arbitrage free and characterized by a constant exogenous recovery rate (r_i). The model further assumes that the US Treasury rate is the default free rate, and that default events and recovery rates are correlated and dependent on the macro economy.

2.1 The Credit Default Swap Valuation Framework

The valuation framework is derived from the no arbitrage, reduced form credit risk approach. The study considers a pure exchange, frictionless economy with a finite horizon $[0, \tau]$ for a fixed $\tau > 0$. Trading can be discrete or continuous and traded are both defaultable and default-free zero coupon bonds of all maturities. The portfolio of bonds serves as the numeraire. The underlying uncertainty in the economy is represented by a filtered probability space $(\Omega, \mathbf{F}, \mathbf{P})$, where Ω is the state space, \mathbf{F} is the σ -algebra representing measurable events, and \mathbf{P} is the empirical probability measure. Information evolves over the trading interval according to the augmented right-continuous complete filtration $\{\mathbf{F}_t : t \in [0, \tau]\}$ generated by $n \geq 1$ independent Brownian motions $\{W_1(t), W_2(t), \dots, W_n(t) : t \in [0, \tau]\}$

initialized at zero. We let $\mathbf{E}(\bullet)$ denote expectation with respect to the probability measure \mathbf{P} .

Given the assumption of no arbitrage and frictionless markets, there exists an equivalent martingale measure \mathbf{Q} (making all the default free and risk zero coupon bond prices martingales), defined by the property that the price (P_t) at date t of a security promising some contingent amount $X \in F_T$ at time $T \geq t$, and paying zero in default, is represented as follows

$$P_t = \mathbf{E}^{\mathbf{Q}} \left[e^{-\int_t^T (r_s + l_s) ds} \mathbf{1}_{\{\tau > T\}} X \mid \mathbf{F}_t \right] \quad (1)$$

Where τ denotes the random occurrence of default ($\tau < T$), characterized as follows,

$$D_t = \mathbf{1}_{\{\tau \leq t\}} = \begin{cases} 1 & \text{if } \tau \leq t \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

$r_s = f(t, t)$ is the risk free short rate process and l_s is the available market liquidity which anecdotally increases with a deterioration in credit quality. So given equation (1), the prevailing spot rate and existing market liquidity the present values of zero coupon bonds are computed by discounting and taking expectations with respects to \mathbf{Q} .

For purposes of this study it will also be assumed that τ has a risk neutral intensity, denoted $\lambda_t^{\mathbf{Q}}$ where $\lambda_t^{\mathbf{Q}} \Delta$ gives the approximate probability of default for this firm over the time interval $[t, t+\Delta]$. Jarrow (2001) used a two factor reduced form model to price credit default swaps under an assumption of a perfectly liquid market, however this study will relax this assumption by inputting a measure of liquidity

directly into the model. So under the usual Cox process the study assumes that the risk neutral intensity process is modeled as $\lambda_t^Q(t, r_t, l_t)$, where $\{r_t : t \in [0, T]\}$ is a vector stochastic process representing the state variable underlying the macro economy and $\{l_t : t \in [0, T]\}$ is the liquidity proxy that measures market liquidity. Additionally, Jarrow *et al* (2001) further suggests that an increase in market risk increases the likelihood of the issuing firm defaulting on its debt obligation. This study further postulates that liquidity decreases with a deterioration of credit quality, hence we would expect that low investment grade and high yield grade securities would have less liquid markets than high investment grade instruments. This would explain why a number of high yield products are illiquid and difficult to trade in debt markets. Secondly, the Cox-process assumption, the \mathbf{F}_t conditional risk-neutral probability of survival to time T is

$$\mathbf{p}_t = \mathbf{E}^Q \left[1_{\{\tau > T\}} \mid \mathbf{F}_t \right] = 1_{\{\tau > t\}} \mathbf{E}^Q \left[e^{-\int_t^T \lambda^Q(s) ds} \mid \mathbf{F}_t \right] \quad (3)$$

Given equations (1) and (3) we can indirectly determine the value of the credit default swap analytically from the discounted value of the swap payment stream to the protection seller as represented by the following expression of credit risky credit protection payments⁷.

⁷ See Jarrow and Yildirim (2001) for a detailed description of the CDS analytical pricing process.

$$P_t 1_{\{t < \tau\}} = \mathbf{E}_t^{\mathbf{Q}} \left(\int_t^T C_T e^{-\int_t^s [r_u - l_u + \lambda_u] du} ds \right) - \mathbf{E}_t^{\mathbf{Q}} \left(\int_t^T \lambda_{(s)} e^{-\int_t^s [r_u - l_u + \lambda_u] du} ds \right) \quad (4)$$

The protection sellers periodic swap payment receipts in equation (4) continue for a fixed period of time $[0, T]$. This payment by the protection buyer will continue until (a) there is a default by the issuer who has the characteristic λ_t^0 hazard function, or (b) maturity of the credit default swap contract. In the event that there is no default the second part of the expression goes to zero. Following Jarrow and Yildirim (2001), this valuation of the swap payments to the default seller is identical to the value of a risky coupon bond making continuous coupon payments per unit of time, with zero recovery rates, less the cost of credit protection. Given the preceding we can manipulate the analytic formulation in (4) to obtain the value of the risky coupon bond displayed in equation (5).

$$P_t = p_t 1_{\{t < \tau\}} = C_T \int_t^T v(t, s, : 0) ds - \mathbf{E}_t^{\mathbf{Q}} \left(\int_t^T \lambda_{(s)} e^{-\int_t^s [r_u - l_u + \lambda_u] du} ds \right) \quad (5)$$

Computationally, this expression satisfies the valuation framework wherein the risk neutral default intensity is positively correlated with interest rates but independent of market liquidity. Market liquidity is inversely related to credit risk, hence as credit quality deteriorates bid-ask spreads will widen.

At time 0, when $P_t = 0$, equation 5 above can be rearranged to give the value of the swap rate as

$$C_T = \frac{\mathbf{E}_t^{\mathbf{P}} \left(\int_t^T \lambda_{(s)} e^{-\int_t^s [r_u - l_u + \lambda_u] du} ds \right)}{\int_0^T v(0, s, : 0) ds} \quad (6)$$

Analytical computations using expression 6 will yield the market liquidity enhanced CDS price. The Jarrow *et al's* (2001) CDS valuation expression is a special case of the expression in equation 6. When $l_t = 0$ the three-factor liquidity enhanced valuation model collapses to the Jarrow *et al* (2001) two-factor framework. For computational ease it is assumed that, for each issuer $i \in \{1, \dots, n\}$ the economy is Markov in the state variables – interest rates and market liquidity (credit default swap bid-ask prices are used to proxy market liquidity). It is further assumed that both r_t^i and l_t^i follows a Ornstein-Uhlenbeck process under \mathbf{P} , in that

$$dr_t^i = \kappa_p^i [\theta_p^i - r_t^i] dt + \sigma^i dW_t^{i,P} \quad (7)$$

$$dl_t^i = \kappa_p^i [\theta_p^i - l_t^i] dt + \sigma^i dW_t^{i,P} \quad (8)$$

Where $dW_t^{i,P}$ is a standard Brownian motion under \mathbf{P} initialized at $W_0 = 0$, and

where θ_p^i , $\kappa_p^i \neq 0$, and $\sigma^i > 0$ are positive constants⁸.

⁸ a_p^i , θ_p^i , σ^i are the term structure coefficients.

The dynamics of the default intensity and recovery rate jointly determine the dynamics of the CDS spread in the reduce form methodology. Reduced form models introduce separate explicit assumptions on the dynamics of both the probability of default and recovery rates. An abundance of research has been devoted to default intensity dynamics unlike the dynamics of the recovery rate. This maybe partly due to the fact that as shown by Houweling and Vorst (2005) the recovery rate has negligible effects on the outcome of credit default swap prices. Generally, reduced form models assume an exogenous recovery rate that is independent of the probability of default. Given the outcome from Houweling and Vorst (2005) the methodology assumes a constant recovery rate for the preceding analyses.

The intensity function is assumed to be linear in the spot rate of interest and market liquidity. The hazard function's evolution is given under the risk neutral probability \mathbf{Q} . Shown below as

$$\lambda_{(t)} = \max [\lambda_{0(t)} + \lambda_1 r_{(t)} - \lambda_2 l_{(t)} 0] \quad (9)$$

Where :

(a) λ_0 is a deterministic function of time, or the implied autonomous risk premia;

(b) λ_1 and λ_2 are constants;

Additionally, it is further assumed that forward rates of all maturities exist and in the continuous case may be defined as;

$$f(t, T) \equiv \frac{-\partial}{\partial T} \log p(t, T). \quad (10)$$

2.2 Parameter Estimates

The sample variance, mean reversion parameter and long term mean is computed using the smoothed forward rate curves previously generated over the sample period. Since we are using cross sectional data, a cubic-spline interpolation procedure is used to generate additional data points.

A: Default Parameter Estimates

Following a popular approach used by the academic literature, the issuer's default intensity can be modeled as following a stochastic Poisson process, characterized by jumps in the process. The study's default parameter estimates were obtained by using non-linear OLS to fit the term structures of default swap quotes to the estimated arbitrage free spot rate evolution. The non-linear regression procedure is implemented using both cross-sectional and time series observations of swap premia.

Given the spot rate parameter estimates of θ , α , and σ from the spot rate evolution process and the term structure of the swap prices, λ_t is inverted, to obtain the parameter estimates (using a sums of squared error minimizing procedure).

$$\lambda_{(t)} = \max [\lambda_{0(t)} + \lambda_1 r_{(t)} - \lambda_2 l_{(t)} \ 0] \quad (11)$$

Where $\lambda_0(t) \geq 0$ a deterministic function of time " t ", is the implied autonomous risk that CDS investors hold when market risk is 0 and markets are fully liquid. λ_1 and λ_2 are constants. In this formulation, the (pseudo) probability of default per unit of time

is assumed to be the maximum of a linear function of the spot rate $r(t)$, market liquidity and zero. The maximum operator is needed in the expression to ensure that the intensity function doesn't become negative.

The intercept of the intensity process is a deterministic function that is restricted to be a constant. Since this model has only three parameters, there will be errors in matching the term structure of default swap quotes. Hence the parameters were chosen to minimize the sum of squared error between the theoretical and market quotes.

B: Recovery Rate Estimates

There are two approaches for the specification of the recovery rate. The first is to consider it as just another parameter, and estimate it from the data along with the other parameters. The second method is to *a priori* fix a value. A number of researchers in the economic literature suggest that although the first method seems preferable, it turns out that it is hard to identify the recovery rate from the data, see Duffee (1999), Duffee and Singleton (1999) and Houweling and Vorst (2005). This may pose a problem for some applications; fortunately this does not affect the pricing of credit default swaps. Houweling and Vorst (2005) found that the pricing of default swap premium is relatively insensitive to the assumed recovery rate. As such this study assumes a constant recovery rate across the observation period.

C: Liquidity Proxy Estimates

It is generally difficult to measure market liquidity, because liquidity is an elusive phenomenon. However a number of recent studies have shown promise using bid-ask spreads as a proxy for liquidity (see for example Tang and Yan (2006), Acharya and Johnson (2005)). Following the popular approach of using bid-ask spreads as a measure of liquidity, the study used CDS bid-ask spreads for the CDS instruments for the period under study⁹. To be able to determine liquidity levels for each credit grade, the procedure is to calculate the measure of liquidity as the “ask” minus the “bid”. The size of the spread from ask to bid prices will differ mainly because of the difference in liquidity of each asset. This spread differential is then divided by the mid price to derive a unit-less bid-ask measure. This unit-less bid-ask measure forms the instantaneous market liquidity proxy of the study. Tang and Yan (2006) illustrated that since bid-ask spreads are a measure of information asymmetry in the CDS market it would thus be a good liquidity measure. For this study pooled estimates were taken across credit classes. Table 3 in section 4 presents the pooled bid-ask spreads across credit grades. It can be seen from the table that bid-ask spreads increase across credit quality indicating that liquidity decreases with a deterioration of credit quality.

⁹ See Tang and Yan (2006) for an elaborate discussion on using bid-ask prices as a liquidity proxy

3. Description of the Data

This section provides a description of the data used in this paper. Credit default swap bid-ask¹⁰ data for the market liquidity proxy estimates were obtained from Bloomberg. JP Morgan credit default swap mid price data was used to compute pricing estimates and the US Treasury rates were obtained from Bloomberg.

The data analyzed is based on weekly observations from January 2nd, 2004 to August 08th, 2006, where $t = \frac{1}{365}, \dots, \bar{t}$. One observation made during this stage of the exercise is the fact that prior to 2002 the CDS market was not as liquid and active as it is currently. Hence there is not an abundance of reasonable data prior to 2003. The data is comprised of a mixture of 32 US dollar denominated AAA, AA, A, BBB, BB and B credit default swaps issued by 32 fortune 500 companies, across several industries chosen to stratify the various industry groupings such as cable/media, financial, insurance, U.S banks, telecom, energy, retail, technology and manufacturing. The CDS data set was obtained from JP Morgan, a leading market maker for credit default swaps, which are spreads over weekly U.S. Treasury quotes. Quotations are available only on days when there is some level of liquidity in the market as evidenced either through trades or by active market making by a dealer. Bloomberg was then used to obtain CDS bid-ask prices, characteristics such as maturity dates, coupon percentages and seniorities. Bloomberg was also used to obtain weekly U.S Treasury, note and bill prices that were needed for the parameter estimation of the spot rate process. The JP Morgan credit default swap dataset is

¹⁰ The bid-ask prices are consensus quotes among market participants regarding the value of the CDS.

comprised of quotes for contracts of maturities 3 through 10 years. During the sample period there are 135 weeks of default swap quotes per reference entity.

In reality since most of the credit default swaps trading activity is within the 5-year time to maturity group, the price quotes on the 5 year CDS premia will be used in the study's pricing analyses. For an issuer to be included in the sample, it must have at least 130 weekly observations of its 5-year CDS data points. As a result of this selection technique, the CDS dataset used in this study covers 41 issuers with an average of 133 weekly observations per issuers, for maturities of 1, 3, 5, 7 and 10 years respectively.

As stated earlier the market liquidity will be derived from bid-ask spreads. In any market that is in equilibrium, there will generally be a difference between the best quoted ask price and the best quote bid price. That difference is called the bid-ask spread (or bid-offer spread). For the market liquidity proxy, this study uses the percentage bid-ask spread, which is the bid-ask premia divided by the mid price. Tang and Yan (2006) suggest that bid-ask spreads measure trading costs that compensate market makers for the risk of adverse selection and hedging costs. Depending upon the market bid-ask quotes may be expressed as actual prices, yields, implied volatilities, etc. The average of the bid and ask prices is called the mid-offer price.

4. Discussion of the Empirical Results

Estimates of the term structure parameters of θ_p^i , κ_p^i and σ^i for the spot interest rate, market observed credit default swap (CDS) prices and the bid-ask liquidity proxy were computed using cross-sectional data of 1-year, 3-year, 5-year, 7-year and the 10-year credit default observations. The sample variance, mean reversion parameter and long term mean is computed using the smoothed forward rate curves previously generated over the sample period. The parameter estimates and the standard errors are presented in table 1a below;

Table 1a: Term Structure Coefficients for spot rate

Parameter	Coefficient	Std Error
κ	0.1400	0.0237
θ	2.2900	0.01737
σ_r	0.0200	0.00037

To estimate the study's three-factor reduced form model the term structure estimates for the liquidity process' volatility, long-term mean and reversion factor were computed from the smoothed CDS bid-ask liquidity forward curve. The estimates and standard error are presented below in table 1b;

Table 1b: Term Structure Coefficients for Liquidity variable

Parameter	Coefficient	Std Error
κ	0.1420	0.0295
θ	6.7632	3.195
σ_r	0.2318	0.029

Looking at the sample's bid ask quotes, it is apparent that the average bid-ask spread increases with deterioration in credit rating. High bid-ask spreads are associated with low liquidity, hence firms perceived to be of a lower risk of default have lower instantaneous bid-ask credit spreads than firms in the high default risk group. Table 2 presents the pooled bid-ask spreads by credit ratings.

Table 2: Pooled 5 Yr CDS Bid-Ask Spreads

Rating	Tranche
	5 Yr
AAA	3.13
AA	4.68
A	5.15
BBB	7.21
BB	9.27
B	8.34

Table 3 summarizes characteristics of the sample's default swap quotes. The table shows the average CDS quotes by industry and credit rating over the observation period for each of the listed referenced entities. Table 3 also shows that the average cross-sectional spread for each entity increases with the maturity of the swap.

Table 4a and 4b, which present the sample's descriptive statistics, both illustrate that the mean spread increases as credit quality deteriorated for the 5yr CDS spread. Also, the mean spreads were found to be higher at longer maturities. This is important because anecdotal evidence suggests that credit rating is an important determinant of default premiums, and as Houweling et al (2005) suggests,

“average premiums move linearly with credit quality”, hence average premium appears to increase with a decrease in credit quality. The pooled means, standard deviations and coefficient of variations presented in table 4a further illustrate the degree of variability underlying the sample data. The estimates indicate variability both within and across credit qualities. Trúck et al (2004) suggests that this variability stems from the speed of reaction of the various financial markets to credit quality changes. The CDS market is perceived to react much quicker to anticipated credit quality changes than the bond or stock markets, see Hull *et al* (2004) and Longstaff *et al* (2005).

4.1 Results of the Two-Factor Model

Table 5 presents the non-linear regression results of the average default swap parameters over the sample period, measured in basis points. The average autonomous parameter estimates for λ_0 ranged from a low of 0.6347 for Cendant Corp to a high of 1110.37 for AMR Corp, whilst the λ_1 estimate ranged from a low of 0.8546 for Arrow Electronics to a high of 245.37 for AMR Corp of Texas. All parameters were found to be statistically different from zero, and with R^2 values above 90%. Since the study used estimated hazard functions derived from the credit default swap premia, it is believed that this will give a fairly good representation of the default and credit risk relationship. From the analysis, and consistent with Jarrow *et al's* (2001) findings, λ_1 is positive indicating that as interest rates increase, the

likelihood of default also increases, an observation that conforms to economic principles. The autonomous credit risk component was also found to be positive which indicates that when interest rates are zero there is a fractional amount of credit spread unaffected by the spot rate.

The hazard rate functions of all 32 firms had root mean errors (RMSE) of less than 0 basis points, with exception of AMR and UNUM Provident. These fitting errors compare well since the root mean square error, a kind of generalized standard deviation, which measures differences between subgroups or relationships between variables is close to or less than zero. These small errors are evidence that the CDS valuation model is relatively successful in capturing both the level and variation in default and credit risks.

Using these estimated parameter values for the hazard function the study then used the closed form expression in equation 6 to solve for the credit default premia. Summary statistics for the difference between the implied and the market credit default swap premia are reported in Table 6. These summary statistics include the average differences with their respective t-statistics and the mean absolute percentage pricing error (the average of the absolute spread error divided by the observed swap premia). From Table 6 the pricing errors range from being positive to slightly negative for the study's reference entities indicating that on average the model does a good job of pricing the CDS premia observed in the market. The t-statistics show that approximately 90 percent of the average differences of the sample are statistically significant.

Although, the average differences are generally all positive (except Altria Group, Alcan, Cendant, Dow Chemical, WAMU, Viacom, AMR, Nordstrom, Marriot and XL Capital), there is significant cross-sectional variation in the average differences across credit rating. For example in the AAA category, the average absolute differences range from low values of 0.17 basis points for GE to 1.00 basis points for XL Capital and in the A category a low of 0.08 basis points for ACE Ltd to 21.2 basis points for Viacom Corp, respectively. The cross-sectional mean and standard deviation of the average differences are 114.96 and 12.54 basis points respectively. This appears consistent with Duffie (1999) who suggests that reduce form models have difficulty explaining the observed term structure of credit spreads across firms of different qualities. In particular, such models have difficulty generating both relatively flat yield spreads when firms have low credit risk and steeper yield spreads when firms have higher credit risk. It is believed that this shortcoming can be overcome by extending the 2-parameter hazard function model to incorporate a parameter that measures market liquidity of the corporate credit market, since it is believed that the level of liquidity in the market place can have a significant effect on prices.

4.2 Results of the Three-Factor Model

Tables 7a and 7b present the parameter estimates for the extended reduced form model discussed in section 2. Bloomberg CDS bid-ask spread measured in

basis points was used as a proxy of market liquidity. Table 7a demonstrates that the average autonomous parameter estimate for λ_0 ranged from a low of 0.584 for Capital One bank to a high of 1522.5 basis points for AMR. The estimates for λ_1 ranged from a low of 0.715 for Marriott Hotels to a high of 177.993 for AMR Corp, whilst the absolute λ_2 estimate ranged from a low of 0.0245 for IBM Corp to a high of 44.8615 for Cendant. All estimates were statistically different from zero, and returned an R^2 that were better than those of the two factor model and above the 90% level.

Consistent with the earlier discussion of the two-factor model results, the root mean error of the estimates were all less than zero, and λ_1 was found to be positive, indicating that as interest rates increase, the likelihood of default also rises. Additionally, the extended model also returned inverse parameter estimates for λ_2 , indicating that market liquidity moves inversely with credit risk. This result is in line with findings by Acharya and Johnson (2005) who also, while using a bid-ask liquidity proxy, found an inverse relationship between credit risk and liquidity. The effects of liquidity across credit quality is demonstrated in table 7b, which affirms the hypotheses that market liquidity does impact CDS valuation, and investors autonomous risks are greater than is apparent in the absence of market liquidity. Table 7b further illustrates that there is significant cross-sectional variation along the lines of credit quality, industry and liquidity. This variation could be due in part to one or a combination of the following observations:

- (a) The CDS market's quick response to anticipated credit quality changes;

(b) The individual CDS level of liquidity or illiquidity¹¹.

Table 8 presents the variance of the implied and market CDS prices, based on the closed form expression in equation 6. The results also include summary statistics such as the t-Stats and the mean absolute percentage pricing error of the differences between the implied and actual CDS prices. As with the earlier discussion of the two-factor model, the pricing errors range from positive to slightly negative. This finding appears to compliment a recent study by Longstaff *et al* (2005) who suggests that the market prices of credit risk may be larger than observed. The cross-sectional standard deviation of 3.93 suggests that although there is the significant variation across credit ratings, the liquidity parameter helps in explaining some of the observed cross-sectional variation seen in the two-factor specification. This finding is supported by prior work done by Tang and Yan (2006) who found that on average liquidity explains about 40% of the cross-sectional variation in CDS spreads. Further, the model's pooled measurement error of -3.50 indicates far better explanatory performance of the extended model over the two-factor credit-market risk model, thereby highlighting the importance of market liquidity in the valuation model. Additional evidence of the liquidity proxy's enhancement of the valuation model is further explained by the mean absolute percentage valuation error¹² which returned 3.72 percent for the three-factor model as compared to the 8.40% of the two-factor model.

¹¹ The CDS level of liquidity or illiquidity¹¹; Chen *et al* (2004) suggests that the liquidity effect in Bond spreads remains significant even after controlling for several yield spread factors such as credit ratings, maturity and tax effects.

¹² This is the average of the absolute valuation error divided by the mean observed CDS price.

Finally, the superior performance of the three-factor model versus the two-factor model bolsters the hypothesis that market liquidity influences the valuation of credit default swaps. This is consistent with results obtained by Chen *et al* (2004) in their work in examining the importance of liquidity in corporate yield spreads. In addition, the study also illustrates that implied autonomous credit risk is larger than investors would be led to believe in the absence of a market liquidity measure. The two factor model did not fully quantify the value of implied autonomous credit risk. However once liquidity was added to the model we saw that investors had more implied exposure than previously perceived.

5. Conclusion

Over the past few years, financial markets have been marked by increased volatility and risk, due in part to the decline of credit quality brought on by unfavorable economic shocks. As a result of this scenario, there has been a sharp rise in the use of credit default swaps by investors to reduce credit and market risks. Given the growing importance of the CDS market in both widening and deepening credit markets, this study attempts to investigate the importance of liquidity in the valuation process of CDS. A liquidity proxy was introduced to a popular two-factor approach to see its effects on the valuation process.

Both models were implemented empirically and the study found clear evidence that (a) The inclusion of liquidity improved the CDS valuation

methodology, (b) The implied cost of autonomous credit risk is significantly higher with the inclusion of a liquidity proxy. The absence of market liquidity underestimates implied autonomous credit risk. The inclusion of the liquidity measure indicated that investors' implied risk was greater because of liquidity's effect through credit quality. The analysis also confirmed that the inclusion of the liquidity parameter did improve the valuation methodology. This was demonstrated through comparisons of the implied premia results of both models, the extended three-factor model performed better in matching the observed market data, suggesting that the addition of the market liquidity variable improved the explanatory power of the model.

Economic theory suggests that market and credit risk are related to each other and not separable. The study found that market liquidity is important in valuing credit default swaps because it affects credit risks indirectly through credit quality. The results of both models in the study have affirmed this view. The empirical results also illustrated levels of cross-sectional variation across both the high grade and yield credit grades. These levels of cross-sectional variation increase in the absence of a liquidity measure.

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Table 3: Summary Statistics showing the average 1 through 10 year spreads of the listed reference entities and their credit ratings used in the study.

Issuer	Ticker	Industry	S&P	Moody's	1Year	3 Year	5 Year	7 Year	10 Year
General Electric	GE	Industrial	AAA	Aaa	9	18	27	33	41
Verizon	VZ	Telecom	A	A3	11	27	39	47	58
Altria	MO	Consumer	BBB+	Baa2	48	73	93	108	126
Aetna	AET	HealthCare	A-	A3	25	29	33	48	55
Ace Insurance	ACE	Insurance	A-	A3	18	35	52	57	63
Alcan	AL	Mining	BBB+	Baa1	14	24	34	37	43
Alcoa	AA	Mining	A-	A2	11	20	29	36	44
Alltel	AT	Telecom	A-	A2	11	28	40	47	57
American Express	AXP	Credit Services	A+	A1	7	17	25	31	37
American International Group	AIG	Insurance	AA	Aa2	10	16	22	28	34
Arrow Electronics	ARW	Electronics/wholesale	BBB-	Baa3	42	70	95	117	130
Bristol-Myers-Squibb	BMJ	Drug Manufacturer	A+	A1	9	20	31	37	44
Cendant	CD	Rental and Leasing	BBB+	Baa1	19	34	49	58	69
Caterpillar	CAT	Industrial	A	A2	11	18	25	32	38
Cingular	AT&T	Telecom	A	Baa1	18	29	40	48	58
Capital One	COF	Credit Services	BBB	A3	26	40	54	62	70
IBM	IBM	Computer	A+	A1	11	16	21	26	32
Wal-Mart	WMT	Consumer/Discount	AA	Aa2	9	11	15	20	25
Target	TGT	Consumer/Discount	A+	A2	13	17	22	28	37
Dow Chemical	DOW	Chemical	A-	A3	16	28	38	46	54
Washinton Mutual Bank	WAMU	Bank	A	A2	14	27	40	45	54
Viacom	VIA	Cable	A	Baa3	13	27	41	51	63
Carnival Corporation	CCL	Entertainment	A-	A3	17	31	45	53	64
American Airlines	AMR	Airline	B	Caa2	2157	2079	2001	1,836	1701
Lucent	LU	Manufacturing	B	B1	152	215	278	297	314
Starwood Resorts	HOT	Lodging	BB+	Ba2	88	113	138	152	159
UNUM Provident Group		Insurance/Benefit	BB+	Ba1	157	187	217	227	240
KB Homes	KBH	Residential Construction	BB	Ba1	117	150	183	188	193
Nordstrom	JWN	Consumer/apparel	A	Baa1	14	24	34	43	54
Halliburton	HAL	Oil and Gas	BBB+	Baa1	36	45	54	65	73
Marriott	MAR	Lodging	BBB+	Baa2	16	29	42	51	60
XL Capital	XL	Insurance	AAA	A3	22	34	46	50	57

Table 4a: Pooled Sample Mean Std Dev and Coefficient of Variation

Credit Rating	Mean	Std Dev	σ/μ
AAA	36.50	5.47	0.15
AA	18.50	4.06	0.22
A	35.06	9.16	0.26
BBB	59.86	14.10	0.25
BB	179.33	46.22	0.26
B	1139.50	205.63	0.22

Table 4b: Descriptive Statistics of Credit Default Spread. The table presents summary statistics for credit default swap premia for the indicated firms making up the study. The spreads are expressed in basis points. N denotes the number of observations, and the remainder of the data includes the individual mean spread, standard deviation, min and max spreads.

Issuer	Ticker	Industry	S&P	Moody's	Tenor	Mean	Min	Max	Std Dev	σ/μ	N
General Electric Capital	GE	Finance	AAA	Aaa	5 Yr	27	19.00	36.00	4.40	0.1629	321
Verizon	VZ	Telecom	A	A3	5 Yr	39	19.00	60.00	10.94	0.2805	321
Altria	MO	Consumer	BBB+	Baa2	5 Yr	105	77.00	125.00	16.45	0.1567	321
Aetna	AET	HealthCare	A-	A3	5 Yr	39	25.00	42.00	10.63	0.2726	321
Ace Insurance	ACE	Insurance	A-	A3	5 Yr	52	33.00	115.00	12.16	0.2339	321
Alcan	AL	Mining	BBB+	Baa1	5 Yr	34	20.00	58.00	10.63	0.3127	321
Alcoa	AA	Mining	A-	A2	5 Yr	29	15.00	46.00	8.11	0.2797	321
Alltell	AT	Telecom	A-	A2	5 Yr	40	20.00	59.00	12.89	0.3222	321
American Express	AXP	Credit Services	A+	A1	5 Yr	25.00	18.00	31.00	4.73	0.1892	321
American International Group	AIG	Insurance	AA	Aa2	5 Yr	22	17.00	52.00	4.77	0.2166	321
Arrow Electronics	ARW	Technology/Computer	BBB-	Baa3	5 Yr	95	64.00	133.00	23.54	0.2478	321
Bristol-Myers-Squibb	BMY	Drug Manufacturer	A+	A1	5 Yr	31	15.00	50.00	9.87	0.3184	321
Cendant	CD	Rental and Leasing	BBB+	Baa1	5 Yr	35	38.00	61.00	5.66	0.1618	321
Caterpillar	CAT	Industrial	A	A2	5 Yr	25	18.00	30.00	3.87	0.1549	321
Cingular	AT&T	Telecom	A	Baa1	5 Yr	40	21.00	70.00	14.44	0.3611	321
Capital One	COF	Credit Services	BBB	A3	5 Yr	54	30.00	75.00	11.33	0.2097	321
IBM	IBM	Computer	A+	A1	5 Yr	21.00	10.00	31.00	4.90	0.2335	321
Wal-Mart	WMT	Retail - Non Food	AA	Aa2	5 Yr	15	8.00	20.00	3.35	0.2232	321
Target	TGT	Retail - Non Food	A+	A2	5 Yr	22	10.00	32.00	6.68	0.3035	321
Dow Chemical	DOW	Chemical	A-	A3	5 Yr	38	17.00	56.00	11.74	0.3090	321
Washinton Mutual Bank	WAMU	Bank	A	A2	5 Yr	40	32.00	53.00	4.19	0.1046	321
Viacom	VIA	Cable/Media	A	Baa3	5 Yr	41	22.00	62.00	10.97	0.2676	321
Carnival Corporation	CCL	Entertainment	A-	A3	5 Yr	45	25.00	70.00	14.30	0.3177	321
American Airlines	AMR	Airline	B	Caa2	5 Yr	2001	1489.00	2837.00	336.39	0.1681	321
Lucent	LU	Manufacturing	B	B1	5 Yr	278	130.00	430.00	74.87	0.2693	321
Starwood Resorts	HOT	Hotels	BB+	Ba2	5 Yr	138	80.00	205.00	40.46	0.2932	321
UNUM Provident Group		Insurance/Benefit	BB+	Ba1	5 Yr	217	101.70	370.60	53.99	0.2488	321
KB Homes	KBH	Residential Construction	BB	Ba1	5 Yr	183	90.00	260.00	44.22	0.2416	321
Nordstrom	JWN	Retail - Non Food	A	Baa1	5 Yr	34	22.00	70.00	6.20	0.1824	321
Halliburton	HAL	Oil and Gas	BBB+	Baa1	5 Yr	54	29.00	88.00	19.57	0.3624	321
Marriott	MAR	Hotels	BBB+	Baa2	5 Yr	42	25.00	56.00	11.51	0.2740	321
XL Capital	XL	Insurance	AAA	A3	5 Yr	46	34.00	75.00	6.54	0.1421	321

Table 5: Parameter Estimates of the Two Factor model - This table presents the parameter estimates and summary statistics from fitting the default parameter model to the CDS data of the indicated reference entities.

Issuer	Ticker	Industry	S&P	Moody's	λ_0	Std Error	λ_1	Std Error	SSE	MSE	N
General Electric	GE	Industrial	AAA	Aaa	1.1511	0.1915	6.9521	0.0524	12.9740	0.0796	331
Verizon	VZ	Telecom	A	A3	29.4445	0.0572	2.8060	0.0156	1.1578	0.0071	331
Altria	MO	Consumer	BBB+	Baa2	54.6608	0.3645	17.3340	0.1073	190.3000	0.7582	331
Aetna	AET	HealthCare	A-	A3	7.6348	0.1000	6.8644	0.0294	14.3324	0.0571	331
Ace Insurance	ACE	Insurance	A-	A3	3.8551	0.1265	13.4868	0.0372	22.9113	0.0913	331
Alcan	AL	Mining	BBB+	Baa1	19.3460	0.1013	3.7594	0.0277	3.6294	0.0223	331
Alcoa	AA	Mining	A-	A2	16.4528	0.0891	3.0594	0.0244	2.8102	0.0172	331
Alltel	AT	Telecom	A-	A2	33.2267	0.0565	2.6432	0.0155	1.1292	0.0069	331
American Express	AXP	Credit Services	A+	A1	12.9567	0.0797	3.8295	0.0218	2.2455	0.0138	331
American International Group	AIG	Insurance	AA	Aa2	13.5958	0.0299	5.1498	0.0163	0.0792	0.0005	331
Arrow Electronics	ARW	Electronics/wholesale	BBB-	Baa3	2.9443	0.8546	0.8546	0.2337	258.2000	1.5844	331
Bristol-Myers-Squibb	BMJ	Drug Manufacturer	A+	A1	4.2996	0.1931	6.4785	0.0528	13.1914	0.0809	331
Cendant	CD	Rental and Leasing	BBB+	Baa1	0.6347	0.1399	28.9499	0.0765	1.7368	0.0107	331
Caterpillar	CAT	Industrial	A	A2	15.1797	0.1210	3.3605	0.0331	5.1804	0.0318	331
Cingular	AT&T	Telecom	A	Baa1	2.6585	0.1896	9.4926	0.0518	12.7102	0.0780	331
Capital One	COF	Credit Services	BBB	A3	36.7961	0.0665	5.9882	0.0182	1.5628	0.0096	331
IBM	IBM	Computer	A+	A1	17.2451	0.0148	1.0652	0.0041	0.0778	0.0005	331
Wal-Mart	WMT	Consumer/Discount	AA	Aa2	5.5751	0.0706	2.5742	0.0193	1.7619	0.0108	331
Target	TGT	Consumer/Discount	A+	A2	8.0629	0.1936	5.4309	0.0529	13.2532	0.0813	331
Dow Chemical	DOW	Chemical	A-	A3	8.5963	0.1921	7.0758	0.0525	13.0521	0.0801	331
Washinton Mutual Bank	WAMU	Bank	A	A2	35.9184	0.0399	1.3590	0.0109	0.5637	0.0035	331
Viacom	VIA	Cable	A	Baa3	19.3129	0.0606	9.0132	0.0332	0.3263	0.0020	331
Carnival Corporation	CCL	Entertainment	A-	A3	3.2065	0.1315	9.9833	0.0360	6.1181	0.0375	331
American Airlines	AMR	Airline	B	Caa2	1110.3720	7.4311	245.3688	2.0318	19527.2000	119.8000	331
Lucent	LU	Manufacturing	B	B1	141.3093	1.1879	35.9131	0.3248	499.0000	3.0611	331
Starwood Resorts	HOT	Lodging	BB+	Ba2	88.8308	0.5698	13.6759	0.1558	114.8000	0.7043	331
UNUM Provident Group		Insurance/Benefit	BB+	Ba1	79.6071	1.6766	46.5657	0.4584	994.0000	6.0982	331
KB Homes	KBH	Residential Construction	BB	Ba1	104.4741	0.4993	21.6705	0.1365	88.1448	0.5408	331
Nordstrom	JWN	Consumer/apparel	A	Baa1	20.2062	0.0874	4.0098	0.0239	2.6988	0.0166	331
Halliburton	HAL	Oil and Gas	BBB+	Baa1	6.3966	0.2910	13.1521	0.0796	29.9462	0.1837	331
Marriott	MAR	Lodging	BBB+	Baa2	39.0514	0.0184	0.8862	0.0050	0.1195	0.0007	331
XL Capital	XL	Insurance	AAA	A3	31.2332	0.1077	4.6646	0.0294	4.1006	0.0252	331

Table 6: Two-Factor model Summary Statistics for the difference between simulated and observed CDS premia - Avg Diff is the difference of the simulated over the actual observed spread.

Issuer	Ticker	Industry	S&P	Moody's	Implied Spread	Avg Diff	Mean Abs % Valuation Error	t-Stat
General Electric	GE	Industrial	AAA	Aaa	20.17	0.17	0.64	0.71
Verizon	VZ	Telecom	A	A3	37.12	7.12	18.26	11.70
Altria	MO	Consumer	BBB+	Baa2	99.93	19.93	21.43	21.77
Aetna	AET	HealthCare	A-	A3	25.56	0.56	1.71	0.95
Ace Ltd	ACE	Insurance	A-	A3	39.08	0.08	0.15	0.12
Alcan	AL	Mining	BBB+	Baa1	29.63	-4.37	12.87	7.39
Alcoa	AA	Mining	A-	A2	30.21	4.21	14.51	9.32
Alltel	AT	Telecom	A-	A2	40.46	0.46	1.15	0.64
American Express	AXP	Credit Services	A+	A1	23.43	0.43	1.74	1.65
American International Group	AIG	Insurance	AA	Aa2	24.73	0.73	3.31	2.75
Arrow Electronics	ARW	Electronics/wholesale	BBB-	Baa3	80.21	14.21	14.95	10.85
Bristol-Myers-Squibb	BMJ	Drug Manufacturer	A+	A1	22.03	2.03	6.53	3.69
Cendant	CD	Rental and Leasing	BBB+	Baa1	63.22	-11.78	24.04	37.38
Caterpillar	CAT	Industrial	A	A2	24.37	1.37	5.50	6.38
Cingular	AT&T	Telecom	A	Baa1	28.63	5.63	14.08	7.01
Capital One	COF	Credit Services	BBB	A3	53.19	22.19	41.10	35.22
IBM	IBM	Computer	A+	A1	20.16	1.16	5.52	4.25
Wal-Mart	WMT	Consumer/Discount	AA	Aa2	12.62	0.62	4.12	3.32
Target	TGT	Consumer/Discount	A+	A2	22.92	4.92	22.38	13.25
Dow Chemical	DOW	Chemical	A-	A3	27.96	-0.04	0.11	0.07
Washington Mutual Bank	WAMU	Bank	A	A2	39.64	-0.36	0.91	1.56
Viacom	VIA	Cable	A	Baa3	38.80	-21.20	51.71	34.73
Carnival Corporation	CCL	Entertainment	A-	A3	30.52	2.52	5.60	3.17
American Airlines	AMR	Airline	B	Caa2	1781.73	-0.27	0.01	0.01
Lucent	LU	Manufacturing	B	B1	239.57	0.57	0.21	0.14
Starwood Resorts	HOT	Lodging	BB+	Ba2	126.25	0.25	0.18	0.11
UNUM Provident		Insurance/Benefit	BB+	Ba1	207.02	58.02	26.74	19.31
KB Homes	KBH	Residential Construction	BB	Ba1	163.77	1.77	0.97	0.72
Nordstrom	JWN	Consumer/apparel	A	Baa1	31.18	-2.82	8.30	8.18
Halliburton	HAL	Oil and Gas	BBB+	Baa1	42.38	8.38	15.52	7.70
Marriott	MAR	Lodging	BBB+	Baa2	41.48	-0.52	1.25	0.82
XL Capital	XL	Insurance	AAA	A3	44.00	-1.00	2.18	2.76
Total Average Diff and Mean Abs % Valuation Error						114.96	8.40%	

Table 7a: Parameter Estimates of the Three Factor model - Presented are the hazard function's parameter results for the study's reference entities

Issuer	Ticker	Industry	S&P	Moody's	λ_0	Std Error	λ_1	Std Error	λ_2	Std Error	SSE	MSE	N
General Electric	GE	Industrial	AAA	Aaa	11.9351	0.8917	5.1890	0.1482	-0.8224	0.0672	17.9730	0.0548	331
Verizon	VZ	Telecom	A	A3	33.1737	0.2451	2.1959	0.0407	-0.2841	0.0185	1.3581	0.0041	331
Altria	MO	Consumer	BBB+	Baa2	15.7214	3.7698	29.6891	0.6266	-3.6600	0.2840	321.2000	0.9793	331
Aetna	AET	HealthCare	A-	A3	14.9797	0.6031	5.6324	0.1024	-0.5340	0.0435	21.9207	0.0436	331
Ace Ltd	ACE	Insurance	A-	A3	14.7832	0.7189	11.6530	0.1221	-0.7941	0.0519	31.1400	0.0619	331
Alcan	AL	Mining	BBB+	Baa1	24.3375	0.4954	2.9436	0.0823	-0.3809	0.0373	5.5469	0.0169	331
Alcoa	AA	Mining	A-	A2	21.4711	0.4143	2.2389	0.0689	-0.3827	0.0312	3.8789	0.0118	331
Alltel	AT	Telecom	A-	A2	36.6100	0.2537	2.0898	0.0422	-0.2578	0.0191	1.4552	0.0044	331
American Express	AXP	Credit Services	A+	A1	17.3110	0.3751	3.1179	0.0623	-0.3322	0.0283	3.1795	0.0097	331
American International Group	AIG	Insurance	AA	Aa2	52.6673	3.2630	2.7820	0.1977	-3.1421	0.2624	0.1084	0.0003	331
Arrow Electronics	ARW	Electronics/wholesale	BBB-	Baa3	12.6063	3.7102	28.7950	0.6167	-3.6243	0.2795	311.1000	0.9486	331
Bristol-Myers-Squibb	BMJ	Drug Manufacturer	A+	A1	14.7461	0.9139	4.7707	0.1519	-0.7967	0.0688	18.8769	0.0576	331
Cendant	CD	Rental and Leasing	BBB+	Baa1	512.7843	74.7083	21.7088	4.5273	-44.8615	6.0088	56.8358	0.1733	331
Caterpillar	CAT	Industrial	A	A2	21.7051	0.5750	2.2937	0.0956	-0.4978	0.0433	7.4736	0.0228	331
Cingular	AT&T	Telecom	A	Baa1	0.7384	0.7989	10.5624	0.1328	-0.8844	0.0602	14.4271	0.0440	331
Capital One	COF	Credit Services	BBB	A3	0.5837	2.0208	15.8081	0.3359	-1.9719	0.1522	92.2950	0.2814	331
IBM	IBM	Computer	A+	A1	17.5643	0.0800	1.0132	0.0133	-0.0245	0.0060	0.1448	0.0210	331
Wal-Mart	WMT	Consumer/Discount	AA	Aa2	9.4754	0.3319	1.9368	0.0552	-0.2977	0.0250	2.4899	0.0076	331
Target	TGT	Consumer/Discount	A+	A2	18.0885	0.9343	3.7924	0.1553	-0.7649	0.0704	19.7307	0.0602	331
Dow Chemical	DOW	Chemical	A-	A3	19.4319	0.8934	5.3041	0.1485	-0.8263	0.0673	18.0403	0.0550	331
Washinton Mutual Bank	WAMU	Bank	A	A2	37.8494	0.1984	1.0441	0.0330	-0.1466	0.0149	0.8895	0.0027	331
Viacom	VIA	Cable	A	Baa3	316.2048	27.5793	28.1641	1.6713	-29.2996	2.2182	7.7455	0.0236	331
Carnival Corporation	CCL	Entertainment	A-	A3	12.4239	0.5326	8.4745	0.0885	-0.7016	0.0401	6.4117	0.0195	331
American Airlines	AMR	Airline	B	Caa2	1522.5250	34.8332	177.9932	5.7900	-31.4364	2.6242	27424.0000	83.6099	331
Lucent	LU	Manufacturing	B	B1	206.0509	5.6091	25.3306	0.9324	-4.9388	0.4226	711.1000	2.1680	331
Starwood Resorts	HOT	Lodging	BB+	Ba2	121.3780	2.6399	8.3553	0.4388	-2.4825	0.1989	157.5000	0.4802	331
UNUM Provident		Insurance/Benefit	BB+	Ba1	36.9341	7.9075	61.8381	1.3144	-7.7142	0.5957	1413.3000	4.3088	331
KB Homes	KBH	Residential Construction	BB	Ba1	135.7713	2.1931	16.5511	0.3645	-2.3848	0.1652	108.7000	0.3314	331
Nordstrom	JWN	Consumer/apparel	A	Baa1	24.3880	0.4247	3.3266	0.0706	-0.3191	0.0320	4.0771	0.0124	331
Halliburton	HAL	Oil and Gas	BBB+	Baa1	23.4243	1.3300	10.3678	0.2211	-1.2982	0.1002	39.9807	0.1219	331
Marriott	MAR	Lodging	BBB+	Baa2	40.0984	0.0857	0.7150	0.0142	-0.0798	0.0065	0.1658	0.0005	331
XL Capital	XL	Insurance	AAA	A3	37.4781	0.4947	3.6435	0.0822	-0.4762	0.0373	5.5314	0.0169	331

Table 7b: Pooled average parameter estimates across credit classes for 2 and 3 factor models

Two Factor Model			Three Facto Model			
Credit Rating	λ_0	λ_1	Credit Rating	λ_0	λ_1	λ_2
AAA	16.19	5.81	AAA	24.71	4.42	-0.65
AA	9.59	3.86	AA	31.07	2.36	-1.72
A	14.89	5.62	A	38.84	5.98	-2.30
BBB	17.53	8.93	BBB	89.94	15.72	-7.98
BB	90.97	27.30	BB	98.03	28.91	-4.19
B	625.84	140.64	B	864.29	101.66	-18.19

Table 8: Three-Factor model Summary Statistics for the difference between simulated and observed CDS premia - Avg Diff is the difference of the simulated over the actual observed spread.

Issuer	Ticker	Industry	S&P	Moodys	Implied Spread	Avg Diff	Mean Abs % Valuation Error	t-Stat
General Electric	GE	Industrial	AAA	Aaa	20.05	0.05	0.18	0.20
Verizon	VZ	Telecom	A	A3	37.08	7.08	18.16	11.63
Altria	MO	Consumer	BBB+	Baa2	63.85	-16.15	17.36	17.64
Aetna	AET	HealthCare	A-	A3	25.67	0.67	2.03	1.14
Ace Insurance	ACE	Insurance	A-	A3	39.24	0.24	0.47	0.36
Alcan	AL	Mining	BBB+	Baa1	29.57	-4.43	13.02	7.48
Alcoa	AA	Mining	A-	A2	24.77	-1.23	4.25	2.73
Alltel	AT	Telecom	A-	A2	40.42	0.42	1.05	0.59
American Express	AXP	Credit Services	A+	A1	23.38	0.38	1.54	1.46
American International Group	AIG	Insurance	AA	Aa2	24.72	0.72	3.28	2.72
Arrow Electronics	ARW	Electronics/wholesale	BBB-	Baa3	64.59	-1.41	1.49	1.08
Bristol-Myers-Squibb	BMJ	Drug Manufacturer	A+	A1	21.91	1.91	6.15	3.47
Cendant	CD	Rental and Leasing	BBB+	Baa1	74.85	-0.15	0.30	0.47
Caterpillar	CAT	Industrial	A	A2	24.30	1.30	5.20	6.03
Cingular	AT&T	Telecom	A	Baa1	23.10	0.10	0.24	0.12
Capital One	COF	Credit Services	BBB	A3	29.25	-1.75	3.24	2.78
IBM	IBM	Computer	A+	A1	20.16	1.16	5.50	4.23
Wal-Mart	WMT	Consumer/Discount	AA	Aa2	12.57	0.57	3.82	3.08
Target	TGT	Consumer/Discount	A+	A2	22.81	4.81	21.85	12.94
Dow Chemical	DOW	Chemical	A-	A3	27.83	-0.17	0.44	0.26
Washinton Mutual Bank	WAMU	Bank	A	A2	39.62	-0.38	0.96	1.65
Viacom	VIA	Cable	A	Baa3	60.42	0.42	1.03	0.69
Carnival Corporation	CCL	Entertainment	A-	A3	30.42	2.42	5.38	3.04
American Airlines	AMR	Airline	B	Caa2	1777.01	-4.99	0.25	0.27
Lucent	LU	Manufacturing	B	B1	238.83	-0.17	0.06	0.04
Starwood Resorts	HOT	Lodging	BB+	Ba2	125.88	-0.12	0.09	0.05
UNUM Provident Group		Insurance/Benefit	BB+	Ba1	149.07	0.07	0.03	0.02
KB Homes	KBH	Residential Construction	BB	Ba1	163.42	1.42	0.77	0.58
Nordstrom	JWN	Consumer/apparel	A	Baa1	31.13	-2.87	8.44	8.32
Halliburton	HAL	Oil and Gas	BBB+	Baa1	42.19	8.19	15.17	7.52
Marriott	MAR	Lodging	BBB+	Baa2	41.46	-0.54	1.27	0.84
XL Capital	XL	Insurance	AAA	A3	43.93	-1.07	2.34	2.95
Total Average Diff and Mean Abs % Valuation Error						-3.50	3.72%	