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Non-Parametric Tests of Real Exchange rates in the Post-Bretton Woods Era

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Abstract

This paper examines the mean-reverting property of real exchange rates. Earlier studies have generally not been able to reject the null hypothesis of a unit-root in real exchange rates, especially for the post-Bretton Woods floating period. The results imply that long-run purchasing power parity does not hold. More recent studies, especially those using panel unit-root tests, have found more favorable results, however. But, Karlsson and Löthgren (2000) and others have recently pointed out several potential pitfalls of panel unit-root tests. Thus, the panel unit-root test results are suggestive, but they are far from conclusive. Moreover, consistent individual country time series evidence that supports long-run purchasing power parity continues to be scarce. In this paper, we test for long memory using Lo's (1991) modified rescaled range test, and the rescaled variance test of Giraitis, Kokoszka, Leipus, and Teyssière (2003). Our testing procedure provides a non-parametric alternative to the parametric tests commonly used in this literature. Our data set consists of monthly observations from April 1973 to April 2001 of the G-7 countries in the OECD. Our two tests find conflicting results when we use U.S. dollar real exchange rates. However, when non-U.S. dollar real exchange rates are used, we find only two cases out of fifteen where the null hypothesis of an unit-root with short-term dependence can be rejected in favor of the alternative hypothesis of long-term dependence using the modified rescaled range test, and only one case when using the rescaled variance test. Our results therefore provide a contrast to the recent favorable panel unit-root test results.

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Non-Parametric Tests of Real Exchange Rates in the Post-Bretton Woods Era

1. Introduction

The theory of purchasing power parity (PPP) occupies a central place in international economics. It is a key building block in the monetary models of exchange rate determination. In the flexible-price monetary model of exchange rate [e.g., Frenkel (1978)], PPP is assumed to hold continuously. In the sticky-price monetary model [e.g., Dornbusch (1976)], because of sticky prices in the short run, PPP does not hold, but is a maintained assumption for the long run. Recent empirical tests of PPP have mainly focused on the long run given that there are frequent large and persistence short-run deviations from PPP. The question of interest is whether deviations from PPP are transitory or permanent. Thus, the empirical tests generally take the form of testing for stationarity of the real exchange rate. If deviations from PPP are transitory, the time series of the real exchange rate is a stationary series. On the other hand, if deviations from PPP are permanent, then the time series of the real exchange rate is non-stationary and contains a unit root.

Until quite recently, empirical results have not been very encouraging, especially using data from the post-Bretton Woods period [see the survey by Rogoff (1996)]. One explanation is that the augmented Dickey-Fuller (ADF, 1981) test, which is one of the most generally used tests for PPP, has low power against plausible alternatives, especially against trend-stationary alternative [see e.g., Hakkio (1986), and DeJong, Nankervis, Savin, and Whiteman (1992)]. Thus, it is not possible to distinguish whether the failure to find PPP is due to the low power of the tests employed or that PPP does not hold in the post-Bretton Woods floating period.

One way to increase the power of the empirical tests is to use longer span of data. Examples are studies by Diebold, Husted, and Rush (1991),[1] Lothian, and Taylor (1996), and Engel and Kim (1999). The results are inconclusive, however. Moreover, Rogoff (1996) and others have noted that long spans of data typically mix fixed and floating exchange rates data, and the economic implications of mixing data from the two exchange rate regimes are unclear. Furthermore, long spans of time series data may potentially contain serious structural breaks. Engel (1996) also argued that these studies can have serious size biases, and may fail to reject a sizable unit root. Finally, these studies also do not shed much light on the question of whether or not PPP is a valid hypothesis in the post-Bretton Woods floating period.

Another way to increase the power of unit-root tests is to use panel data. Recent examples using data from the post-Bretton Woods floating exchange rate period include studies by Jorion and Sweeney (1996), Papell (1997), Papell and Theodoridis (1998), Koedijk, Schotman, and Van Dijk (1998), Sarno and Taylor (1998), O'Connell (1998). Once again, the results are mixed and are not very robust. For example, Papell (1997) found that evidence in favor of long-run PPP is dependent on the size of the panel and the countries included, while Rogoff (1996) noted that the evidence of long-run PPP tends to be much more favorable when high inflation countries are included. Finally, Karlsson and Löthgren (2000), using Monte Carlo simulations, found that for panels with long spans of data, the null hypothesis of unit roots could be erroneously rejected even when only a small proportion of the series is stationary. For panels with short spans of data, however, Karlsson and Löthgren (2000) found that the null hypothesis of unit roots is frequently not rejected even when a large fraction of the series is stationary. Thus, they concluded that the rejection or the non-rejection of the null hypothesis of unit roots in panel unit root tests do not provide sufficient evidence to conclude that all the series in the panels are stationary or that they all have a unit root.

In this paper, we test for long memory in real exchange rate, to be defined in the next section, using data from the post-Bretton Woods period. We use two non-parametric tests in our study. The first is the modified rescaled range test proposed by Lo (1991), and Haubrich and Lo (2001), and the second is the rescaled variance test of Giraitis, Kokoszka, Leipus, and Teyssière (2003). Our paper is motivated by several factors. First, even-though there is a growing body of literature that supports long-run PPP for the post-Bretton Woods period, the results do not appear to be robust, and consistent individual country time series evidence from that time period continues to be elusive. Second, long-memory processes allow for more flexibility in modeling and testing for long-run PPP than the ADF test, but have received relatively little attention in the literature. Third, non-parametric tests for long-run PPP offer an alternative to parametric tests, given the problems associated with parametric tests. That is, we are interested in knowing what non-parametric tests can tell us about the time series behavior of real exchange rates. This has hardly been explored in the literature on long-run PPP, and unit-root test in general. Fourth, beyond the obvious implications that our empirical results may have for exchange rate theories, a more compelling reason to examine real exchange rates as long-memory processes is that they may arise as a natural consequence of dynamic aggregation. This possibility has not been systematically examined in the literature, and we will do this in Section 3. Fifth, while there is some evidence to support Lothian's (1998) assertion that the frequent failures to find favorable evidence for long-run PPP in the post-Bretton Woods period may be due to the use of the U.S. dollar as the base currency, to our knowledge, this too has never been investigated systematically. We provide such a systematic study using seven base currencies. Finally, as we will also make clear in the next section, our study also has implications for what Rogoff (1996) has called "the purchasing power parity puzzle".

The rest of this paper is structured as follows. In Section 2, we first discuss long-memory time series and its implications for the "purchasing power parity puzzle". We next discuss the sources of long memory in real exchange rates in Section 3. We discuss the modified rescaled range and rescaled variance tests in Section 4. In Section 5, we discuss our data set and present our empirical results. Section 6 contains our summary and conclusions.

2. Fractional Differencing and Long Memory Time Series

In this paper, we define the real exchange rate in its natural logarithm form as:

$$r_t = e_t + P_t^* - P_t, \tag{1}$$

where e_t is the natural logarithm of the nominal exchange rate, defined as the domestic currency price of one unit of foreign currency, P_t is the natural logarithm of an index of the domestic price level, and P_t^* is the natural logarithm of an index of the foreign price level. A common modeling strategy of unit-root tests is to model the real exchange rate as an autoregressive integrated moving-average process of order (p, d, q), denoted as ARIMA (p, d, q), i.e.,

$$\Phi(L)(1-L)^d r_t = \Theta(L)\varepsilon_t, \tag{2}$$

or, more generally, its autoregressive approximation:

$$\Psi(L)(1-L)^d r_t = \varepsilon_t, \tag{3}$$

where $\Phi(L) = 1 - \phi_1 L - ... - \phi_p L^p$, $\Theta(L) = 1 + \theta_1 L + ... + \theta_q L^q$, L is the lag operator, ε_t is a white noise process, and d is the degree of differencing. All roots of $\Phi(L)$ and $\Theta(L)$ are assumed to lie outside the unit circle, and $\Psi(L) = \dfrac{\Phi(L)}{\Theta(L)}$. If $d = 1$, the real exchange rate is a unit-root process. Thus, for long run PPP to hold, it requires that $d = 0$. The condition that a stationary series requires that $d = 0$ is viewed by many economists to be arbitrary and too restrictive. Granger and Joyeux (1980), and Hosking (1981) have proposed the autoregressive fractionally integrated moving-average process (ARFIMA) as a generalization to the ARIMA model. In an ARFIMA model, d is allowed to assume values in the set of real numbers, as opposed to only integer values as in the conventional ARIMA model. For $|d| < 0.5$, the ARFIMA process is stationary and invertible, for $d \in [0.5, 1)$, the process is still mean reverting but the variance of r_t in equation (3) is infinite.[2] When $d = 1$ or $d = 0$, the ARFIMA process reduces to a standard ARIMA $(p, 1, q)$ process or a stationary ARMA $(p, 0, q)$ process, respectively. Thus, a wider range of mean-reverting behavior can be modeled by allowing for fractional differencing.

One of the characteristics of a fractionally-differenced time series is that its autocorrelation function decays hyperbolically, rather than at an exponential rate as in a stationary ARMA process. This non-negligible long-range dependence between observations spaced far apart led statisticians to call this class of models long-memory models. This point is important for the debate on the "purchasing power parity puzzle" as defined by Rogoff (1996). The puzzle centers on the difficulties in reconciling the observed extremely high short-term volatility of real exchange rates with a slow rate of convergence to PPP following a shock. Conventional wisdom appears to suggest that the speed of parity reversion following a shock has a half-life of about 3 – 5 years [see Rogoff (1996), Cheung and Lai (2000), and Murray and Papell (2002)]. While monetary shocks combined with sticky nominal prices or wages can provide an explanation for the short-run volatility of real exchange rates, the estimated 3 – 5 years for the shocks to damp out appear to be more consistent with real rather than monetary shocks.[3] This half-life of 3 – 5 years is calculated from estimating standard ARMA models. If, however, real exchange rates follow ARFIMA processes, the speed of parity reversion will most likely be different from the current estimated half-life of 3 – 5 years. Thus, knowledge of whether real exchange rates are stationary ARMA processes or ARFIMA processes can contribute to our understanding of the process of parity reversion following a shock. It is worth emphasizing that the long-range dependence in the fractionally-differenced processes relates to the slow decay of the autocorrelation functions and not to any permanent effect.

Despite its apparent flexibility and advantages over the ARIMA models [see Sowell (1992)], long-memory models are not widely used in Economics. A number of statistical techniques, surveyed in Baillie (1996), are available for estimating long-memory models. In this paper, we test for long memory in real exchange rates using two non-parametric tests. The first is the modified rescaled range test of Lo (1991), and the second is the rescaled variance test of Giraitis et al (2003).[4] In addition to providing an alternative to the more generally used parametric tests (e.g., the ADF tests) for long-run PPP, non-parametric tests also have several advantages over conventional parametric tests. For example, non-parametric tests allow for very general functional form of the model. Thus, we are not restricted to linear models as with the ADF tests. This is especially important since some recent studies have found evidence of non-linear mean-reversion of real exchange rates [see the survey by Sarno and Taylor (2002)]. Moreover, the error structure of the model need not be a Gaussian process, more general processes are allowed, including ARCH processes, which are common in financial time series. The non-parametric tests that we use, however, do not produce an estimate of the fractional differencing parameter, d. This is of only secondary importance to us, however, since our main focus here is to determine whether shocks to real exchange rates are permanent or transitory.

3. Long Memory Processes in Real Exchange Rates

Before proceeding to a discussion of our empirical technique, we will discuss several potential sources of long memory in real exchange rates. First, it should be noted that if shocks to real exchange rates follow ARFIMA processes, then real exchange rates would also follow ARFIMA processes. Second, Cheung (1993b) has provided some empirical evidence of long memory in nominal exchange rates, thus potentially providing another source of long memory in real exchange rates. Third, a more compelling reason is that long memory in real exchange rates is a natural consequence of dynamic aggregation. The explanation follows naturally from the definition of the real exchange rate, Equation (1). The theory is due to Granger (1980) and is discussed in detail both in Granger (1980) and Haubrich and Lo (2001). Simply put, Granger (1980) has shown that, under fairly general conditions, aggregation of dynamic equations can lead to a class of models that can be well approximated by ARFIMA processes. In the real exchange rate model, the index of the price level from Equation (1) can be thought of as being given approximately as:

$$P_t = \sum_{i=1}^{N} \pi_i p_{i,t}, \quad \sum_{i=1}^{N} \pi_i = 1, \tag{4}$$

where π_i, and $p_{i,t}, i = 1, \ldots, N$ are the weights and components of the price index, respectively. Suppose each component price is generated according to the following ARMA(1, 0) process:[5]

$$p_{i,t} = \delta_i p_{i,t-1} + e_{i,t}, i = 1, \ldots, N, \tag{5}$$

and $e_{i,t}, i = 1, \dots, N$ are white-noise processes. The aggregation and averaging of the component prices would result in the price index being an ARMA(*N, N-1*) processes, unless there are cancellations of the AR roots. In the U.S., the consumer price index (CPI) has well over several thousands component prices, resulting in an extremely high order ARMA process for the CPI, at least in theory. However, this is not observed in practice, and the reason, according to Granger (1980) is that the extremely high order ARMA process for the CPI could be parsimoniously approximated by an ARFIMA process. Following this line of reasoning, if the indexes of domestic and foreign price levels could be approximated by ARFIMA processes, and in light of the empirical evidence provided by Cheung (1993b) on the nominal exchange rates, we can expect real exchange rates to also follow an ARFIMA process.

4. Empirical Methodologies

We start the discussion of our empirical methodologies with the rescaled range statistic, also known as the R/S statistic.[6] The R/S statistic has a long history. It was originally proposed by the English hydrologist Hurst (1951) and subsequently refined by many others, including, for example, Mandelbrot (1972), and Mandelbrot and Wallis (1969). Compared to other techniques of detecting long-range dependence, we probably know more about the rescaled range statistic from the many Monte Carlo simulation studies that have been done. Examples are Mandelbrot and Wallis (1969), Wallis and Matalas (1970), and Davis and Harte (1987). Lo's (1991) modified R/S statistic corrects two shortcomings of the Hurst-Mandelbrot R/S statistic by allowing for (unspecified) short-term dependence and heteroscedasticity. The former makes the modified R/S statistic robust with respect to processes with short-term dependency without having to know their exact forms. The second modification is important when the modified R/S test is applied to financial time series such as exchange rates, where it is well known that such time series tend to exhibit high short-term volatility. Monte Carlo simulations of the size and power of the modified R/S test in finite samples have been performed by Lo (1991), Cheung (1993a), and Haubrich and Lo (2001), and have been shown to be robust against a variety of alternatives.

The modified R/S statistic is given in Lo (1991) as:

$$Q_n(q) \equiv \frac{1}{\hat{\sigma}_n(q)}[Max_{1\le k\le n}\sum_{j=1}^{k}(x_j-\bar{x}) - Min_{1\le k\le n}\sum_{j=1}^{k}(x_j-\bar{x})], \tag{6}$$

where $\hat{\sigma}_n^2(q) = \hat{\sigma}_x^2 + 2\sum_{j=1}^{q}\omega_j(q)\hat{\gamma}_j$, (7)

$$\omega_j(q) \equiv 1 - \frac{j}{q+1}, 1 \le q < n, \tag{8}$$

and $\{x_t\}$ is a stationary time series of sample size n, with sample mean $\bar{x}$, sample variance $\hat{\sigma}_x^2$, and sample autocovariance at lag j given by $\hat{\gamma}_j$.[7] The numerator of equation (6) is the range of the cumulative deviations of x_j from its sample mean, normalized by its heteroskedasticity and autocorrelation consistent variance estimator. The weights, $\omega_j(q)$, is suggested by Newey and West (1987) and is always positive, and q is known as the bandwidth parameter. The difference between Lo's modified R/S and the Hurst-Mandelbrot R/S statistics is in the normalization used. The Hurst-Mandelbrot R/S statistic uses the sample variance, $\hat{\sigma}_x^2$, to normalize the range, i.e., $q = 0$. Since partial sums of white noise constitute a random walk, and $Q_n(q)$ grows without bound as n increases, $Q_n(q)$ is further normalized to

$$V_n(q) = \frac{Q_n(q)}{\sqrt{n}}. \tag{9}$$

The null hypothesis is that $\{x_t\}$ is a short memory process, against the alternative that it is a long memory process. The null hypothesis includes stationary ARMA models, but, as pointed out by Haubrich and Lo (2001), it does not contain the trend-stationary model, however. Finally, it should be noted that the alternative hypothesis is also consistent with ARFIMA models. The limiting distribution of $V_n(q)$ is derived and reported in Lo (1991). The 95% confidence interval with equal probabilities in both tails is [0.809, 1.862].

The second test statistic that we use is the rescaled variance test, or the V/S statistic proposed by Giraitis et al. (2003) and is given as:

$$M_n(q) = \frac{V\hat{a}r(S_1^*, \dots, S_n^*)}{n\hat{\sigma}_n^2(q)}, \tag{10}$$

where $S_k^* = \sum_{j=1}^{k}(x_j - \bar{x})$ are the partial sums of $\{x_t\}$, and $V\hat{a}r(S_1^*, \dots, S_n^*) = \frac{1}{n}\sum_{j=1}^{n}(S_j^* - \bar{S}^*)$ is their sample variance. According to Giraitis et al. (2003), the V/S test is more suitable for time series that exhibit high volatility. In the Monte Carlo study performed by Giraitis et al. (2003), the V/S test is shown to be less sensitive to the choice of the bandwidth parameter q , and achieve a better balance of size and power than Lo's modified R/S statistic. For the V/R test, the asymptotic distribution of $M_n(q)$ coincides with the limiting distribution of the standard Kolmogorov statistic, as demonstrated in Giraitis et al. (2003).

An opened question is how best to determine the bandwidth parameter q . As discussed in Lo (1991), q must increase, but at a slower rate, with the sample size. But, in another context, Lo and MacKinlay (1989) have shown through Monte Carlo simulations that when q becomes too large relative to the sample size, the finite-sample properties of the estimator can be quite different from its asymptotic limit. On the other hand, q cannot be too small such that it fails to include non-negligible higher-order autocorrelations. These considerations suggest that q should be chosen together with the sample data at hand. In the next section, we discuss our data set, the selection of the bandwidth, and present our empirical results.

5. Data Set and Empirical Results

The source of our data is the OECD G-7 countries, obtained online from SourceOECD. Our data consist of monthly observations from April 1973 to April 2001 for the G-7 countries.[8] The G-7 countries are the U.S., the U.K., Canada, Germany, Italy, France, and Japan. In all cases, we use the seasonally adjusted consumer price index as our measure of the average price level. Nominal exchange rates are monthly averages, and the only bilateral nominal exchange rate available on SourceOECD uses the U.S. dollar as the base currency, i.e., foreign currency per U.S. dollar. Because we are also interested in whether the use of non-U.S. dollar based real exchange rates may produce different results, as other studies have found [see Papell (1997), and Papell and Theodoridis (1998) for recent examples], we have also computed real exchange rates based on the pound sterling, the Canadian dollar, the German mark, the Italian Lira, the French franc, and the Japanese yen. These non-U.S. dollar based real exchange rates are computed as cross-rates.[9] This gives us a sample of twenty-one real exchange rates. For comparison purpose, however, we report the results for forty-two real exchange rates (six real exchange rates for each of the seven currencies), but keep in mind that there are only actually twenty-one different real exchange rates. There are also some questions on the appropriateness of using U.S. dollar based real exchange rates. The main reason is the recent findings of Papell (2002) who demonstrated that there were two equal but offsetting breaks in the mean of the U.S. dollar real exchange rates in the 1980s. Our two tests do not take account of this type of structural breaks, and thus the results may be unreliable. Nevertheless, we have decided to include the results with the U.S. dollar based real exchange rates, since, as we will show below, our two tests provide interesting but conflicting results.

As we mentioned in the last section, the choice of q is an opened question. For each real exchange rate, we decide to compute and report the modified R/S and the V/S statistics for q values at 0, 1, 6, 12, and 18. Note that $q = 0$ corresponds to the Hurst-Mandelbrot R/S statistics, denoted by $V_n(0)$. In all cases, however, the calculated statistics are rather stable in this range of q values. Haubrich and Lo (2001) also suggested using Andrews's (1991) data-dependent procedure for determining the optimal bandwidth parameter q. Andrews's procedure sets the optimal q to the integer value of

$$\bar{q} \equiv \left[\frac{3\hat{\alpha}n}{2}\right]^{1/3}, \quad \hat{\alpha} \equiv \frac{4\hat{\rho}^2}{\left(1-\hat{\rho}^2\right)^2}, \tag{11}$$

where $\hat{\rho}$ is the first-order autocorrelation coefficient. The optimal bandwidth selected according to Equation (11) for all real exchange rate series is less than the maximum reported q = 18. We also take the first difference of the real exchange rate, i.e., $x_t = \Delta r_t$ before applying the tests. Tables 1 - 7 report our empirical results, the results for the optimal bandwidth are reported in the last column of each table.

Starting first with the Hurst-Mandelbrot R/S statistics, in all seven tables, the null hypothesis is not rejected at the 95% confidence interval. Thus, according to the Hurst-Mandelbrot R/S statistics, there is no statistical evidence that any of the twenty-one real exchange rates is mean-reverting over the post-Bretton Woods sample period. Turning next to the modified R/S statistics, starting with Table 1 where the U.S. dollar is the base currency, we see that the null hypothesis is not rejected for all six of the real exchange rates at the 95% confidence interval. We obtain the same results in Table 2, using the Canadian dollar as the base currency; in Table 6 using the Italian lira as the base currency; and Table 7 using the pound sterling as the base currency. The Japanese yen real exchange rate results in Table 3 are interesting. For q = 1, the null hypothesis is not rejected in all cases. For q =6, 12, and 18, the null hypothesis is rejected for the franc/yen real exchange rate, and rejection of the null hypothesis occurs for the mark/yen real exchange rate at q = 6, 12. These two cases with the Japanese yen real exchange rate point out the importance of allowing for short-term dependence in the rescaled range statistic, and also the importance of choosing the bandwidth q. The results in Table 4 where the French franc is the base currency, and Table 5 where the German mark is the base currency show a very similar pattern. As already noted, except for the Japanese yen real exchange rate, the null hypothesis of a unit-root process with short-term memory is not rejected for the reported values of q for the remaining five real exchange rates at the 95% confidence interval. Finally, the results using the optimal bandwidth gives us only two rejections out of twenty-one, that of franc/yen and mark/yen real exchange rates.

Taken together, the empirical results with the modified R/S test reported in Tables 1 – 7 are not encouraging for long-run PPP, since only two of twenty-one real exchange rates appear to be mean-reverting. Several other results are also worth noting. First, contrary to Lothian's (1998) assertion, our results suggest that the negative evidence for long-run PPP cannot be attributed to the use of the U.S. dollar as the base currency. For example, using the Canadian dollar, or the pound sterling, or the Italian lira as the base currency all produced identical negative evidence for long-run PPP. Second, there is some evidence to suggest that the use of the Japanese yen and not the German mark does produce some marginally favorable evidence for long-run PPP. Third, our results suggest that the modified R/S statistics are generally quite stable, especially beyond $q = 4$ in most cases. The stability of the modified R/S statistics strengthens our conclusions.

Turning now to the V/S test statistics, we see that there are differences compared with the modified R/S test, especially with using the U.S. dollar as the base currency. In Table 1, the results suggest that the null hypothesis could be rejected for the Canadian dollar/U.S. dollar, mark/dollar, and lira/dollar real exchange rates, and marginally for the franc/dollar real exchange rate. Beyond that, the mark/lira real exchange rate (in Tables 5 and 6) is the only other real exchange rate where the null hypothesis could be rejected. Interestingly, while the null hypothesis is rejected using the modified R/S test for the franc/yen, and mark/yen rates, it is not rejected by the V/S test. Taken together, and ignoring the U.S. dollar based real exchange rate results for the moment, we do not find strong evidence in favor of long-run PPP.

There are two possible explanations for the conflicting results with the U.S. dollar based real exchange rates. First, the Monte Carlo simulation study of Cheung (1993a) has showed that the modified R/S statistic is biased in favor of finding long-term dependence when there is a mean shift. The breaks in the U.S. dollar real exchange rates in the 1980s appear to be two equal off-setting mean breaks [see Papell (2002)], the modified R/S test may have low power against this type of structural breaks. On the other hand, the structural breaks during this period caused the U.S. dollar real exchange rates to be more volatile than other periods in our sample [see Lothian (1998)]. The V/S test may be capturing this increase in volatility without capturing the structural breaks. Thus, rendering the results with the V/S test also unreliable. In sum, while the results with the U.S. dollar based real exchange rates are interesting but puzzling, no definite conclusion could be drawn without further research on the power of the modified R/S and V/S tests against this type of structural breaks. This is left to future research.

6. Summary And Conclusions

Recent empirical studies on long-run PPP appear to have turned the tide on the mostly negative findings that have dominated the literature for a long while. Nevertheless, consistent individual country evidence from the post-Bretton Woods floating exchange rate period continues to be scarce. In this study, we offer an alternative testing procedure to the parametric tests commonly found in the literature. We use the non-parametric tests of Lo (1991), and Giraitis et al. (2003) to study long memory in real exchange rates. We examined twenty-one real exchange rates using monthly data from the post-Bretton Woods floating exchange rate period. Taken together, and excluding the results from using the U.S. dollar as the base currency, the evidence in favor of long-run PPP is marginal at best. The results with the U.S. dollar based real exchange rates are interesting and puzzling, but the results should be interpreted with great caution. Our results, however, are consistent with the results of Baum, Barkoulas, and Caglayan (1999). They estimated ARFIMA processes for real exchange rates in the post-Bretton Woods era using Sowell's (1992) maximum likelihood procedure, and found almost no evidence to support long-run PPP. Second, our empirical evidence suggests that the mostly negative evidence against long-run PPP cannot be attributed to the use of the U.S. dollar as the base currency as Lothian (1998) has suggested when using data from this period. There is some evidence to suggest that the use of the Japanese yen as the base currency produces some marginally better results for long-run PPP. We find our results both surprising and somewhat disappointing. We are surprised in light of the encouraging results from recent empirical studies, especially from panel studies, and also because our empirical procedures allowed for more flexibility in modeling mean-reverting behaviors in real exchange rates. We are somewhat disappointed because we continue to look for solid empirical support for long-run PPP to anchor our theoretical models of exchange rate determination.

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Table 1
Base currency is U.S. dollar

	$q = 0$	$q = 1$	$q = 6$	$q = 12$	$q = 18$	$q = \overline{q}$
Canada:						
$V_n(q)$	1.726	1.620	1.523	1.365	1.290	(3) 1.563
$M_n(q)$	0.170*	0.150*	0.132*	0.105*	0.095*	(3) 0.139*
Japan:						
$V_n(q)$	1.486	1.295	1.170	1.127	1.109	(6) 1.170
$M_n(q)$	0.099*	0.075*	0.061	0.057	0.055	(6) 0.061
France:						
$V_n(q)$	1.571	1.395	1.245	1.196	1.164	(5) 1.371
$M_n(q)$	0.118*	0.093*	0.074	0.068	0.065	(5) 0.075*
Germany:						
$V_n(q)$	1.676	1.473	1.376	1.311	1.264	(5) 1.371
$M_n(q)$	0.119*	0.092*	0.080*	0.073	0.068	(5) 0.080*
Italy:						
$V_n(q)$	1.753	1.517	1.362	1.317	1.295	(6) 1.362
$M_n(q)$	0.132*	0.099*	0.080*	0.075*	0.072	(6) 0.080*
U.K.:						
$V_n(q)$	1.530	1.322	1.226	1.216	1.242	(6) 1.226
$M_n(q)$	0.070	0.053	0.046	0.045	0.047	(6) 0.046

Notes: * denotes that the null hypothesis can be rejected at the 95% confidence interval. For the modified R/R test, the 95% confidence interval with equal probabilities at both tails is [0.809, 1.862]. The source is Lo (1991). For the R/V test, the critical value at the 95% confidence level is 0.074.
$\overline{q}$ denotes the optimal bandwidth selected using Equation (11). The number in parenthesis is the optimal value.

Table 2
Base currency is Canadian dollar

	$q = 0$	$q = 1$	$q = 6$	$q = 12$	$q = 18$	$q = \overline{q}$
U.S.:						
$\boldsymbol{V_n(q)}$	1.726	1.620	1.523	1.365	1.290	(3) 1.563
$\boldsymbol{M_n(q)}$	0.170*	0.150*	0.132*	0.105*	0.095*	(3) 0.139*
Japan:						
$\boldsymbol{V_n(q)}$	1.246	1.092	0.989	0.957	0.910	(6) 0.989
$\boldsymbol{M_n(q)}$	0.072	0.055	0.045	0.043	0.039	(6) 0.045
France:						
$\boldsymbol{V_n(q)}$	1.338	1.172	0.994	0.957	0.956	(6) 0.994
$\boldsymbol{M_n(q)}$	0.089*	0.068	0.049	0.045	0.045	(6) 0.045
Germany:						
$\boldsymbol{V_n(q)}$	1.436	1.253	1.120	1.072	1.053	(6) 1.120
$\boldsymbol{M_n(q)}$	0094*	0.072	0.057	0.052	0.050	(6) 0.057
Italy:						
$\boldsymbol{V_n(q)}$	1.152	0.990	0.864	0.854	0.883	(6) 0.864
$\boldsymbol{M_n(q)}$	0.072	0.053	0.040	0.039	0.042	(6) 0.040
U.K.:						
$\boldsymbol{V_n(q)}$	1.496	1.299	1.155	1.143	1.186	(6) 1.155
$\boldsymbol{M_n(q)}$	0.069	0.052	0.041	0.040	0.043	(6) 0.041

Notes: See notes to Table 1.

Table 3
Base currency is Japanese yen

	$q = 0$	$q = 1$	$q = 6$	$q = 12$	$q = 18$	$q = \bar{q}$
U.S.:						
$V_n(q)$	1.486	1.295	1.170	1.127	1.109	(6) 1.170
$M_n(q)$	0.099*	0.075*	0.061	0.057	0.055	(6) 0.061
Canada:						
$V_n(q)$	1.246	1.092	0.989	0.957	0.910	(6) 0.989
$M_n(q)$	0.072	0.055	0.045	0.043	0.039	(6) 0.045
France:						
$V_n(q)$	0.980	0.842	0.722*	0.697*	0.748*	(6) 0.722*
$M_n(q)$	0.049	0.036	0.027	0.025	0.029	(6) 0.027
Germany:						
$V_n(q)$	0.970	0.835	0.751*	0.741*	0.811	(6) 0.751*
$M_n(q)$	0.049	0.037	0.030	0.029	0.035	(6) 0.030
Italy:						
$V_n(q)$	1.275	1.090	0.931	0.882	0.924	(7) 0.919
$M_n(q)$	0.055	0.040	0.029	0.026	0.029	(7) 0.029
U.K.:						
$V_n(q)$	1.341	1.166	0.986	0.946	0.951	(6) 0.986
$M_n(q)$	0.082*	0.062	0.044	0.041	0.041	(6) 0.044

Notes: See notes to Table 1.

Table 4
Base currency is French franc

	$q = 0$	$q = 1$	$q = 6$	$q = 12$	$q = 18$	$q = \overline{q}$
U.S.:						
$\boldsymbol{V_n(q)}$	1.571	1.395	1.245	1.196	1.164	(5) 1.371
$\boldsymbol{M_n(q)}$	0.118*	0.093*	0.074*	0.068	0.065	(5) 0.075*
Canada:						
$\boldsymbol{V_n(q)}$	1.338	1.172	0.994	0.957	0.956	(6) 0.994
$\boldsymbol{M_n(q)}$	0.089*	0.068	0.049	0.045	0.045	(6) 0.045
Japan:						
$\boldsymbol{V_n(q)}$	0.980	0.842	0.722*	0.697*	0.748*	(6) 0.722*
$\boldsymbol{M_n(q)}$	0.049	0.036	0.027	0.023	0.029	(6) 0.027
Germany:						
$\boldsymbol{V_n(q)}$	1.354	1.167	1.112	1.034	1.058	(6) 1.112
$\boldsymbol{M_n(q)}$	0.050	0.037	0.034	0.029	0.030	(6) 0.034
Italy:						
$\boldsymbol{V_n(q)}$	1.368	1.248	1.227	1.239	1.245	(4) 1.225
$\boldsymbol{M_n(q)}$	0.085*	0.071	0.069	0.070	0.071	(4) 0.068
U.K.:						
$\boldsymbol{V_n(q)}$	1.290	1.127	1.073	1.129	1.152	(6) 1.073
$\boldsymbol{M_n(q)}$	0.075*	0.057	0.052	0.058	0.060	(6) 0.052

Notes: See notes to Table 1.

Table 5
Base currency is German mark

	$q = 0$	$q = 1$	$q = 6$	$q = 12$	$q = 18$	$q = \overline{q}$
U.S.:						
$V_n(q)$	1.676	1.473	1.376	1.311	1.264	(5) 1.371
$M_n(q)$	0.119*	0.092*	0.080*	0.073	0.068	(5) 0.080*
Canada:						
$V_n(q)$	1.436	1.253	1.120	1.072	1.053	(6) 1.120
$M_n(q)$	0094*	0.072	0.057	0.052	0.050	(6) 0.057
Japan:						
$V_n(q)$	0.970	0.835	0.751*	0.741*	0.811	(6) 0.751*
$M_n(q)$	0.049	0.037	0.030	0.029	0.035	(6) 0.030
France:						
$V_n(q)$	1.354	1.167	1.112	1.034	1.058	(6) 1.112
$M_n(q)$	0.050	0.037	0.034	0.029	0.030	(6) 0.034
Italy:						
$V_n(q)$	1.461	1.288	1.317	1.279	1.267	(5) 1.302
$M_n(q)$	0.102*	0.079*	0.083*	0.078*	0.077*	(5) 0.081*
U.K.:						
$V_n(q)$	1.477	1.281	1.162	1.129	1.121	(6) 1.162
$M_n(q)$	0.099*	0.075*	0.061	0.058	0.057	(6) 0.061

Notes: See notes to Table 1.

Table 6
Base currency is Italian lira

	$q = 0$	$q = 1$	$q = 6$	$q = 12$	$q = 18$	$q = \overline{q}$
U.S.:						
$V_n(q)$	1.753	1.517	1.362	1.317	1.295	(6) 1.362
$M_n(q)$	0.132*	0.099*	0.080*	0.075*	0.072	(6) 0.080*
Canada:						
$V_n(q)$	1.152	0.990	0.864	0.854	0.883	(6) 0.864
$M_n(q)$	0.072	0.053	0.040	0.039	0.042	(6) 0.040
Japan:						
$V_n(q)$	1.275	1.090	0.931	0.882	0.924	(7) 0.919
$M_n(q)$	0.055	0.040	0.029	0.026	0.029	(7) 0.029
France:						
$V_n(q)$	1.368	1.248	1.227	1.239	1.245	(4) 1.225
$M_n(q)$	0.085*	0.071	0.069	0.070	0.071	(4) 0.068
Germany:						
$V_n(q)$	1.461	1.288	1.317	1.279	1.267	(5) 1.302
$M_n(q)$	0.102*	0.079*	0.083*	0.078*	0.077*	(5) 0.081*
U.K.:						
$V_n(q)$	1.322	1.165	1.134	1.211	1.298	(5) 1.120
$M_n(q)$	0.089*	0.069	0.067	0.075*	0.085*	(5) 0.063

Notes: See notes to Table 1.

Table 7
Base currency is pound sterling

	$q = 0$	$q = 1$	$q = 6$	$q = 12$	$q = 18$	$q = \overline{q}$
U.S.:						
$\boldsymbol{V_n(q)}$	1.530	1.322	1.226	1.216	1.242	(6) 1.226
$\boldsymbol{M_n(q)}$	0.070	0.053	0.046	0.045	0.047	(6) 0.046
Canada:						
$\boldsymbol{V_n(q)}$	1.496	1.299	1.155	1.143	1.186	(6) 1.155
$\boldsymbol{M_n(q)}$	0.069	0.052	0.041	0.040	0.043	(6) 0.041
Japan:						
$\boldsymbol{V_n(q)}$	1.341	1.166	0.986	0.946	0.951	(6) 0.986
$\boldsymbol{M_n(q)}$	0.082*	0.062	0.044	0.041	0.041	(6) 0.044
France:						
$\boldsymbol{V_n(q)}$	1.290	1.127	1.073	1.129	1.152	(6) 1.073
$\boldsymbol{M_n(q)}$	0.075*	0.057	0.052	0.058	0.060	(6) 0.052
Germany:						
$\boldsymbol{V_n(q)}$	1.477	1.281	1.162	1.129	1.121	(6) 1.162
$\boldsymbol{M_n(q)}$	0.099*	0.075*	0.061	0.058	0.057	(6) 0.061
Italy:						
$\boldsymbol{V_n(q)}$	1.322	1.165	1.134	1.211	1.298	(5) 1.120
$\boldsymbol{M_n(q)}$	0.089*	0.069	0.067	0.075*	0.085*	(5) 0.063

Notes: See notes to Table 1.

Footnotes

[1] Diebold, Husted, and Rush also modeled their real exchange rate as an autoregressive fractionally integrated moving-average process (ARFIMA), in addition to using long span of data. Thus, it is not possible to tell whether their finding of favorable evidence of long run PPP is due to the long span of data or due to the statistical method used.

[2] Note that mean reversion can occur even when x_t is not covariance stationary.

[3] A recent paper by Murray and Papell (2002) has shown that the half-life estimates are extremely unreliable, however.

[4] Baum, Barkoulas, and Caglayan (1999) estimated ARFIMA processes for real exchange rates with data from the post-Bretton Woods era using Sowell's (1992) exact maximum likelihood method. We choose to use a different estimation method for several reasons. First, not much is known about the size and power, especially in finite samples, of Sowell's estimation method. Second, we view our paper as complimentary to the paper by Baum, Barkoulas, and Caglayan, providing an indirect check of robustness of their results.

[5] Here we assume that the processes are independent. The theory could be extended to handle dependent processes easily.

[6] The R stands for range, and S is the standard deviation of the time series. Thus, R/S is the range normalized by the standard deviation.

[7] There are additional but rather general assumptions regarding the error process. These are stated in Haubrich and Lo (2001) as conditions A1 – A4.

[8] Several countries in the sample belong to the Euro Zone starting in 1999. We are able to find bilateral exchange rates with the U.S. for these countries after 1999, allowing us to extend the data sample to April 2001.

[9] This assumes cross-rate equality except for transaction costs. This is probably a valid assumption for the G-7 countries. Alternatively, as long as the measurement error is a stationary process, our tests for unit-root will not be affected.