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Can a Newly Proposed Mechanism for Allocating Contracts in U.S. Electricity Wholesale Markets Lead to Lower Prices? A Game Theoretic Analysis

Vicki Knoblauch
University of Connecticut

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University of Connecticut

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341 Mansfield Road, Unit 1063
Storrs, CT 06269–1063
Phone: (860) 486–3022
Fax: (860) 486–4463
http://www.econ.uconn.edu/

Abstract

This study of the wholesale electricity market compares the cost-minimizing performance of the auction mechanism currently in place in U.S. markets with the performance of a proposed replacement. The current mechanism chooses an allocation of contracts that minimizes a fictional cost calculated using pay-as-offer pricing. Then suppliers are paid the market clearing price. The proposed mechanism uses the market clearing price in the allocation phase as well as in the payment phase. In concentrated markets, the proposed mechanism outperforms the current mechanism even when strategic behavior by suppliers is taken into account. The advantage of the proposed mechanism increases with increased price competition.

Journal of Economic Literature Classification: C72, D44, L10, L94

Keywords: strategic behavior, multi-unit auction, electricity, Bertrand competition

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1. Introduction:

In the U.S. electricity wholesale market, independent system operators (ISOs) run daily auctions to allocate contracts to suppliers of electric power. Each such auction functions as follows. First, suppliers submit offers to the ISO in the form of energy price curves and start-up costs. Second, the ISO allocates contracts to suppliers in such a way as to minimize costs as calculated from the offer sheets. Finally, suppliers are paid for the energy ordered. However each supplier is paid not the offered price, but the market clearing price. Examples below will illustrate the process.

Much of the discussion to date involving multi-unit auctions has focused on comparing the efficiency and revenue consequences of auction mechanisms with different final payment methods (for example pay at offered price and pay at the market clearing price). See for example Ausubel and Cramton (2002), Hortacsu (2002) and Kahn, Cramton, Porter and Tabors (2001). However, in wholesale electricity multi-unit auctions there are two distinct components to the mechanism which need to be considered; the algorithm an ISO uses to assign contracts to generators and the final payment method. This paper takes as given the current payment method, pay-at-MCP, and compares the cost-minimizing performances of two algorithms; the algorithm in current use which assigns contracts so as to minimize total offered cost and a new proposed method, which assigns contracts so as to minimize the actual final payment which is based on the market clearing price.

It has been pointed out by Yan and Stern (2000) that current auctions as described above do not minimize procurement costs, since they assign contracts that minimize costs calculated from offer sheets, while suppliers are actually paid the market clearing price (MCP). This motivated Luh et al. (2003) to develop an algorithm that assigns contracts that minimize actual procurement costs. Henceforth the term "pay-as-offer formulation" will describe the algorithm currently used by ISO's in electricity auctions and "pay-at-MCP formulation" will describe auctions that assign contracts that minimize actual procurement costs. It is most important to keep in mind that the phrase "pay-as-offer" in "pay-as-offer formulation" refers to the objective function minimized as does "pay-at-MCP" in "pay-at-MCP formulation," but in both formulations, actual payment is made at MCP. See Luh et al. (2003) for the difficulties involved and methods used in creating an algorithm to implement a pay-at-MCP formulation.

The argument that pay-at-MCP formulation will generate procurement costs that are sometimes lower and never higher than those generated by the pay-as-offer formulation because the pay-at-MCP formulation minimizes the correct objective function fails to take into account the effects of strategic behavior by suppliers. In particular, if suppliers tailor their offers to the type of auction they face, it is no longer obvious that the pay-at-MCP formulation will generate lower procurement costs. Studies by Alonso, Trias, Gaitau and Alba (1999) and Vázquez, Rivier and Peŕez (2002) which compare the two auction mechanisms, also fail to consider strategic behavior by suppliers.

In this paper we compare the procurement-cost-minimizing performances of the two formulations, taking into account strategic behavior by the suppliers.

Of course the above is a simplified description of the electricity wholesale market. For example, ISOs run day ahead auctions and same day auctions; grid network constraints affect markets; and markets vary from region to region. In addition, ISOs run retail auctions in which utility companies and other load bearing entities submit demand bids. For an overview of U.S. electricity markets, see Hunt (2002).

In Section 2 an example is presented of a simple market with one ISO and two suppliers, one of which has two strategies: price low and price high. In this one-person game, counterintuitively, the pay-at-MCP formulation generates *higher* procurement costs than the pay-as-offer formulation.

In Section 3 we show that the above example is an anomaly. For markets with two suppliers, the pay-at-MCP formulation generates lower procurement costs than the pay-as-offer formulation more often than it generates higher procurement costs.

In Section 4, we show that as price competition increases the advantage of the proposed algorithm increases. To model competition a third supplier, identical to one of the existing suppliers, is added to the market and consequently the suppliers engage in Bertrand competition. In this framework the pay-at-MCP formulation generates procurement costs that are sometimes lower and never higher than those generated by the pay-as-offer formulation. Studying increased competition in small markets is a proxy for studying large markets. The direct study of large markets with strategic bidding by suppliers is an intractable problem, due to the complexity of the pay-at-MCP and pay-as-offered algorithms.

There are other studies of strategic behavior by suppliers in wholesale electricity markets. For example Spear (2003) studies strategic behavior by suppliers from a general equilibrium perspective. In a study by Le Coq (2002) suppliers precommit to capacity levels, then compete in a pay-at-MCP formulation auction. Suppliers are found to have an incentive to withhold capacity.

Supatgiat, Zhang and Birge (2001) present an algorithm that produces Nash equilibria (profiles of mutual best response offers) for the currently used pay-as-offer formulation. We will not use this algorithm in our work below because we will restrict ourselves to simple markets where the algorithm is not needed. We restrict ourselves to simple markets because 1) there is no known algorithm to produce Nash equilibria for the pay-at-MCP formulation in non-simple markets and 2) in simple markets it is possible to compare the cost-minimizing performance of equilibria for the two formulations in *all* markets of a certain type, and then determine which formulation outperforms the other. We can then gain some idea of how the two formulations compare in larger markets by introducing competition into smaller markets.

In short, a theoretical study of concentrated markets that takes into account strategic behavior by suppliers provides evidence favoring the adoption by ISOs of the pay-at-MCP formulation for wholesale electricity auctions.

Finally, we compare the pay-as-offer formulation with the pay-at-MCP formulation because they are, respectively, the wholesale electricity auction mechanism in current use and a proposed mechanism for which an implementing algorithm has already been constructed. Procurement cost minimization performance has been chosen as the basis for comparison, since 1) cost minimization is the most obvious goal of supply auctions (just as revenue maximization is the most obvious goal of demand auctions), 2) low procurement costs are reflected in low prices to consumers which in the electric power market is one measure of social welfare, 3) under the Federal Power Act The Federal Energy Regulatory Commission's main goal "is to achieve wholesale electricity markets that produce just and reasonable prices and work for customers" (FERC, 2003), 4) Although deregulation of some industries has resulted in lower prices to customers, there have been no such savings in the electricity market (Apt, 2005). Future work could compare the two algorithms on the basis of other social goals such as efficiency, fairness and security.

2. Example 1: A Simple Wholesale Electricity Market Auction.

Consider a wholesale electricity market with two suppliers and one ISO. Supplier 1 has start-up cost zero and two strategies, offer low (O_l) and offer high (O_h). Supplier 1 can supply 0, 1 or 2 units of energy, $O_l(1) = O_l(2) = 25$ dollars per unit and $O_h(1) = O_h(2) = 40$. Supplier 2 has start-up cost 20, and can supply one unit and $O_1(1) = 10$. The ISO's demand is inelastic and equal to 2.

In Game 1, the ISO uses the pay-as-offer formulation to allocate contracts. In Game 2 the ISO uses the pay-at-MCP formulation. These are one-person games, since Supplier 1 is the only party with more than one strategy. These two 1-person games could be combined into one two-person game, but our goal is to compare the outcomes of two auctions, not find the outcome of one more complex auction.

Game 1: Pay-as-offer Formulation.

The ISO calculates procurement costs as the minimum of the pay-as-offer cost of buying 1 unit from each supplier and the pay-as-offer cost of buying 2 units from Supplier 1.

If Supplier 1 submits O_l, the ISO calculates procurement cost as

$\min\{20 + 10 + 25, 2(25)\} = 2(25) = 50$ and Supplier 1's payoff is 50.

If Supplier 1 submits O_h, the ISO calculates procurement costs as

$\min\{20 + 10 + 40, 2(40)\} = 20 + 10 + 40 = 70$ and Supplier 1's payoff is 40.

Therefore in equilibrium Supplier 1 offers low, the MCP is 25 and the actual procurement cost is 50.

Notice that in calculating Supplier 1's payoffs, the generation costs have not been subtracted from 1's revenues. In other words it has been assumed that generation costs are such that subtracting them off does not change the ordinal relationships between payoffs.

This assumption keeps the example simple and avoids breaking the seven cases in the next section into multiple subcases. Generation costs will not be ignored in Section 4, where they can be handled without undue complication.

Game 2: Pay-at-MCP Formulation.

The ISO calculates procurement costs as the minimum of the pay-at-MCP cost of buying 1 unit from each supplier and the pay-at-MCP cost of buying 2 units from Supplier 1.

If Supplier 1 submits O_l, the ISO calculates procurement cost as

$\min\{20 + 25 + 25, 2(25)\} = 2(25) = 50$ and Supplier 1's payoff is 50.

If Supplier 1 submits O_h, the ISO calculates procurement costs as

$\min\{20 + 40 + 40, 2(40)\} = 2(40) = 80$ and Supplier 1's payoff is 80.

Therefore in equilibrium Supplier 1 offers high, the MCP is 40 and the actual procurement cost is 80.

Comparing Games 1 and 2, counterintuitively the pay-at-MCP formulation generates higher procurement costs, despite minimizing the correct objective function. In Section 3 Example 1 is shown to be an anomaly. In Section 4 it will be seen that when a competitor for Supplier 1 is added to the market, no analog of Example 1 exists.

3. Simple Two-Supplier Auctions.

In order to see if Example 1 is typical, we will examine the simplest two-supplier auctions. First we will examine Games 3 and 4 which are identical with Games 1 and 2 respectively, except that the start-up cost for Supplier 2 is $S > 0$, Supplier 2's offer is $O_2(1) = A > 0$ and Supplier 1's strategies are $O_l(1) = O_l(2) = L > 0$ and $O_h(1) = O_h(2) = H > L$.

The solutions to Games 3 and 4 break naturally into four cases.

Case 1. $S + A < L$.

Game 3. Pay-as-offer formulation.

$$O_l\text{: } \min\{S + A + L,\ 2L\} = S + A + L \qquad \pi_1(O_l) = L$$
$$O_h\text{: } \min\{S + A + H,\ 2H\} = S + A + H \qquad \pi_1(O_h) = H$$

1 submits O_h and procurement cost is $S + 2H$.

Game 4. Pay-at-MCP formulation.

$$O_l\text{: } \min\{S + 2L,\ 2L\} = 2L \qquad \pi_1(O_l) = 2L$$
$$O_h\text{: } \min\{S + 2H,\ 2H\} = 2H \qquad \pi_1(O_h) = 2H$$

1 submits O_h and procurement cost is $2H$.

In Case 1, procurement cost is lower in the pay-at-MCP formulation.

Case 2. $L < S + A < H$ and $2L < H$.

Game 3. Pay-as-offer formulation.

$$O_l\text{: } \min\{S + A + L,\ 2L\} = 2L \qquad \pi_1(O_l) = 2L$$
$$O_h\text{: } \min\{S + A + H,\ 2H\} = S + A + H \qquad \pi_1(O_h) = H$$

1 submits O_h and procurement cost is $S + 2H$.

Game 4. Pay-at-MCP formulation.

$$O_l\text{: } \min\{S + \max\{2A,\ 2L\},\ 2L\} = 2L \qquad \pi_1(O_l) = 2L$$
$$O_h\text{: } \min\{S + 2H,\ 2H\} = 2H \qquad \pi_1(O_h) = 2H$$

1 submits O_h and procurement cost is $2H$.

In Case 2, procurement cost is lower in the pay-at-MCP formulation.

Case 3. $L < S + A < H$ and $2L > H$.

Game 3. Pay-as-offer formulation.

$$O_l\text{: } \min\{S + A + L,\ 2L\} = 2L \qquad \pi_1(O_l) = 2L$$
$$O_h\text{: } \min\{S + A + H,\ 2H\} = S + A + H \qquad \pi_1(O_h) = H$$

1 submits O_l and procurement cost is $2L$.

Game 4. Pay-at-MCP formulation.

$$O_l\text{: } \min\{S + \max\{2A,\ 2L\},\ 2L\} = 2L \qquad \pi_1(O_l) = 2L$$
$$O_h\text{: } \min\{S + 2H,\ 2H\} = 2H \qquad \pi_1(O_h) = 2H$$

1 submits O_h and procurement cost is $2H$.

In Case 3, the procurement cost is lower in the pay-as-offer formulation.

Case 4. $H < S + A$.

Game 3. Pay-as-offer formulation.

$$O_l\text{: } \min\{S + A + L,\ 2L\} = 2L \qquad \pi_1(O_l) = 2L$$
$$O_h\text{: } \min\{S + A + H,\ 2H\} = 2H \qquad \pi_1(O_h) = 2H$$

1 submits O_h and procurement cost is $2H$.

Game 4. Pay-at-MCP formulation.

$$O_l\text{: } \min\{S + \max\{2A,\ 2L\},\ 2L\} = 2L \qquad \pi_1(O_l) = 2L$$
$$O_h\text{: } \min\{S + \max\{2A,\ 2H\},\ 2H\} = 2H \qquad \pi_1(O_h) = 2H$$

1 submits O_h and procurement cost is $2H$.

In Case 4, the procurement cost is the same for both formulations.

The less likely knife-edge cases such as $S + A = L$ have been omitted.

Before we assess the results, we should examine the other simple two-supplier auctions. In Game 5 and 6 the player with non-zero start-up costs has two strategies. Supplier 1's start-up cost is $S > 0$, 1's strategies are $O_l(1) = L > 0$ and $O_h(1) = H > L$, and $O_2(1) = O_2(2) = A > 0$. Game 5 is a pay-as-offer formulation auction and Game 6 is a pay-at-MCP formulation auction.

Case A. $S + H < A$.

Game 5. Pay-as-offer formulation.

$$O_l\text{: } \min\{S+L+A,\ 2A\} = S+L+A \qquad \pi_1(O_l) = L$$
$$O_h\text{: } \min\{S+H+A,\ 2A\} = S+H+A \qquad \pi_1(O_h) = H$$

1 submits O_h and procurement cost is $S + 2A$.

Game 6. Pay-at-MCP formulation.

$$O_l\text{: } \min\{S+2A,\ 2A\} = 2A \qquad \pi_1(O_l) = 0$$
$$O_h\text{: } \min\{S+2A,\ 2A\} = 2A \qquad \pi_1(O_h) = 0$$

1 submits O_l or O_h and procurement cost is $2A$.

In Case A, procurement cost is higher in the pay-as-offer formulation.

Case B. $S + L < A < S + H$.

Game 5. Pay-as-offer formulation.

$$O_l\text{: } \min\{S+L+A,\ 2A\} = S+L+A \qquad \pi_1(O_l) = L$$
$$O_h\text{: } \min\{S+H+A,\ 2A\} = 2A \qquad \pi_1(O_h) = 0$$

1 submits O_l and procurement cost is $S + 2A$.

Game 6. Pay-at-MCP formulation.

$$O_l\text{: } \min\{S+2A,\ 2A\} = 2A \qquad \pi_1(O_l) = 0$$
$$O_h\text{: } \min\{S+\ \max\{2A,\ 2H\},\ 2A\} = 2A \qquad \pi_1(O_l) = 0$$

1 submits O_l or O_h and procurement cost is $2A$.

In Case B, procurement cost is higher in the pay-as-offer formulation.

Case C. $A < S + L$.

Game 5. Pay-as-offer formulation.

$$O_l\text{: } \min\{S+L+A,\ 2A\} = 2A \qquad \pi_1(O_l) = 0$$

$$O_h\text{: } \min\{S+H+A,\ 2A\} = 2A \qquad \pi_1(O_h) = 0$$

1 submits O_l or O_h and procurement cost is $2A$.

Game 6. Pay-at-MCP formulation.

$$O_l\text{: } \min\{S+\ \max\{2A,\ 2L\},\ 2A\} = 2A \qquad \pi_1(O_l) = 0$$

$$O_h\text{: } \min\{S+\ \max\{2A,\ 2H\},\ 2A\} = 2A \qquad \pi_1(O_l) = 0$$

1 submits O_l or O_h and procurement cost is $2A$.

In Case C, procurement costs are the same in both formulations.

In summary, in determining the procurement costs in the simplest two-supplier markets there are seven cases to be considered. In four of these cases the procurement cost is higher under the pay-as-offer formulation, and in only one case, the case corresponding to Example 1, is the procurement cost higher under pay-at-MCP formulation.

Even if we consider only Cases 1-4, corresponding to Games 3 and 4, which were constructed using Example 1 as a template, the pay-at-MCP formulation outperforms the pay-as-offer formulation two cases to one.

The outcome of Example 1 is atypical in small, simple markets.

Next it will be shown that when a competitor for the two-strategy supplier is introduced into Games 3 and 4, there are no cases in which the pay-at-MCP formulation generates higher procurement cost. This indicates that in a market with several suppliers the likely increase in competition over that in a small market will lead to an increase in the pay-at-MCP formulation advantage over the pay-as-offer formulation.

4. An Added Competitor.

Some facts about a generalized version of Bertrand competition will be useful in our discussion of a three-supplier wholesale electricity market.

Consider the following two player game. Fix $N > L$. For $i = 1, 2$ let

$$\pi_i(p_1, p_2) = \begin{cases} G(p_i) & \text{if } p_i < p_{-i} \text{ and } p_i < N\ ; \\ g(p_i) & \text{if } p_i = p_{-i} < N; \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

where G: $\Re \to (-\infty, B]$ and g: $\Re \to (-\infty, B]$ for some $B > 0$, $G(p) < 0$ and $g(p) < 0$ if $p < L$, $G(p) = g(p) = 0$ if $p = L$, $G(p) > 0$ and $g(p) > 0$ if $L < p < N$, and if $L < p < N$ then there exists $p' < p$ such that $G(p') > g(p)$.

Proposition 1. *(L, L) is the unique Nash equilibrium of the generalized Bertrand competition game defined by (1).*

Proposition 1 follows from Theorem 2 of Baye and Morgan (2002), since in their proof of that theorem, the role of their condition c), left lower semicontinuity of G, can be played by our condition relating G and g.

Now we consider a wholesale electricity market with three suppliers and one ISO. Supplier 1 has start-up cost zero; has a continuum of strategies: offer price p dollars per unit, p real; can supply 1 or 2 units of energy; and faces generation costs L dollars per unit, $L > 0$. Supplier 2 is identical to Supplier 1. Supplier 3 has start-up cost $S > 0$; can supply 1 unit of energy; and has one strategy: offer price $A > 0$. There is an $N > L$ such that the ISO demands 2 units if the MCP for 2 units is less than N, 1 unit if the MCP for 2 units is greater than or equal to N and the MCP for 1 unit is less than N, and zero otherwise.

Game 7. Pay as offer formulation

$$\pi_i(p_1, p_2) = \begin{cases} p_i - L & \text{if } p_i < p_{-i} \text{ and } S + A < p_i < N \ ; \\ 3(p_i - L)/2 & \text{if } p_i < p_{-i} \text{ and } S + A = p_i < N \ ; \\ 2(p_i - L) & \text{if } p_i < p_{-i},\ p_i < N \text{ and } p_i < S + A \ ; \\ (p_i - L)/2 & \text{if } p_i = p_{-i} \text{ and } S + A < p_i < N \ ; \\ 5(p_i - L)/6 & \text{if } p_i = p_{-i} \text{ and } p_i = S + A < N \ ; \\ p_i - L & \text{if } p_i = p_{-i},\ p_i < N \text{ and } p_i < S + A \ ; \\ 0 & \text{otherwise} \end{cases}$$

This game is a generalized Bertrand competition as defined by (1) and the conditions following (1). Therefore (L, L) is the unique Nash equilibrium. The allocation of contracts depends on the relationship between $S + A$ and L. If $S + A < L$, then the ISO orders one unit from Supplier 3, 1/2 unit from Supplier 1 and 1/2 unit from Supplier 2. The procurement cost is $S + 2L$. If $L < S + A$ then the ISO orders 1 unit from Supplier 1 and 1 unit from Supplier 2. The procurement cost is $2L$.

Game 8. Pay-at-MCP formulation.

$$\pi_i(p_1, p_2) = \begin{cases} 2(p_i - L) & \text{if } p_i < p_{-i} \text{ and } p_i < N \ ; \\ p_i - L & \text{if } p_i = p_{-i} < N; \\ 0 & \text{otherwise} \end{cases}$$

This game is a generalized Bertrand competition. Therefore the only Nash equilibrium is (L, L). The ISO orders 1 unit from Supplier 1 and 1 unit form Supplier 2. The Procurement cost is $2L$.

In summary, if $S + A < L$, then the procurement cost is higher under the pay-as-offer formulation; and if $L < S + A$, then the procurement cost under the pay-as-offer formulation is equal to the procurement cost under the pay-at-MCP formulation. In other words, when Supplier 1 has a competitor, there is no analog to Example 1. The pay-at-MCP formulation outperforms the pay-as-offer formulation.

Concluding Remarks.

In this paper we took the current final payment method, pay at the market clearing price, as given in day-ahead electricity auctions and compared the procurement cost minimizing performance of the current algorithm mechanism used by ISO's (pay-as-offer) and a proposed mechanism (pay-at-MCP). First an example was presented in which, counterintuitively, the pay-at-MCP formulation generated *higher* procurement costs than the pay-as-offer formulation. Next, we showed that the above example is an anomaly. For markets with two suppliers, the pay-at-MCP formulation generates lower procurement costs than the pay-as-offer formulation more often than it generates higher procurement costs. Next, we showed that as competition increases the performance of the proposed algorithm improves. When there is price competition even in small markets the pay-at-MCP formulation generates procurement costs that are sometimes lower and never higher than those generated by the pay-as-offer formulation.

As a final comment, some ISOs pay only part of the start-up costs associated with the suppliers chosen. Since our Example 1 is driven by start-up costs, the pay-at-MCP formulation performance advantage documented above will be even greater for those ISOs that pay only part of start-up costs.

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