

9-1-2007

# Does the Federal Reserve Follow a Non-Linear Taylor Rule?

Kenneth Petersen  
*University of Connecticut*

Follow this and additional works at: [http://digitalcommons.uconn.edu/econ\\_wpapers](http://digitalcommons.uconn.edu/econ_wpapers)

---

## Recommended Citation

Petersen, Kenneth, "Does the Federal Reserve Follow a Non-Linear Taylor Rule?" (2007). *Economics Working Papers*. Paper 200737.  
[http://digitalcommons.uconn.edu/econ\\_wpapers/200737](http://digitalcommons.uconn.edu/econ_wpapers/200737)

This is brought to you for free and open access by the Department of Economics at DigitalCommons@UConn. It has been accepted for inclusion in Economics Working Papers by an authorized administrator of DigitalCommons@UConn. For more information, please contact [digitalcommons@uconn.edu](mailto:digitalcommons@uconn.edu).



University of  
Connecticut

*Department of Economics Working Paper Series*

**Does the Federal Reserve Follow a Non-Linear Taylor Rule?**

Kenneth Petersen  
University of Connecticut

Working Paper 2007-37

September 2007

---

341 Mansfield Road, Unit 1063  
Storrs, CT 06269-1063  
Phone: (860) 486-3022  
Fax: (860) 486-4463  
<http://www.econ.uconn.edu/>

This working paper is indexed on RePEc, <http://repec.org/>

## **Abstract**

The Taylor rule has become one of the most studied strategies for monetary policy. Yet, little is known whether the Federal Reserve follows a non-linear Taylor rule. This paper employs the smooth transition regression model and asks the question: does the Federal Reserve change its policy-rule according to the level of inflation and/or the output gap? I find that the Federal Reserve does follow a non-linear Taylor rule and, more importantly, that the Federal Reserve followed a non-linear Taylor rule during the golden era of monetary policy, 1985-2005, and a linear Taylor rule throughout the dark age of monetary policy, 1960-1979. Thus, good monetary policy is associated with a non-linear Taylor rule: once inflation approaches a certain threshold, the Federal Reserve adjusts its policy-rule and begins to respond more forcefully to inflation.

**Journal of Economic Literature Classification:** E4, E5

**Keywords:** Taylor rule, Federal Reserve, non-linearity, monetary policy

I would like to thank Christian Zimmermann (adviser), Paul

# 1 Introduction

The Taylor rule is a linear algebraic interest rate rule that specifies how the Federal Reserve must adjust its federal funds rate to the inflation rate and the output-gap. This simple interest rate rule characterizes a monetary policy strategy for achieving the objectives of monetary policy: price stability and maximum employment. Though simple, this linear interest rate rule represents an optimal policy-rule, under the condition that the Federal Reserve is minimizing a symmetric quadratic loss function and that the aggregate supply function is linear. See, for instance, Svensson (1997) and Clarida et al. (1999, 2000).

Both theoretical and empirical reasons exist, however, to suggest that the Federal Reserve may be following a non-linear Taylor rule. Firstly, if the Federal Reserve is minimizing an asymmetric loss function in which negative and positive inflation- and output-gap deviations are, respectively, assigned different weights, then a non-linear Taylor rule is optimal. See, for instance, Nobay and Peel (2003), Ruge-Murcia (2003), Dolado et al. (2005) and Surico (2006).

Furthermore, the output-gap and inflation are inherently non-linear processes with asymmetric adjustment mechanisms. For example, over the business cycle output exhibits short and sharp recessions, but long and smooth recoveries. Similarly, inflation increases more rapidly than it decreases over the business cycle. See, for example, Hamilton (1989), Beaudry and Koop (1993), van Dijk and Franses (1999) and Neftci (2001). In such an environment, the Federal Reserve has to respond differently to negative versus positive inflation and output-gap shocks, to bring inflation and output back to target. In short, a non-linear Taylor rule is necessary.

A non-linear time series model is needed to explain non-linear policy behavior. Several non-linear time series models are available to choose from; the artificial neural network (ANN) model, the Markov-switching model, and the smooth transition regression (STR) model. The ANN model can fit the in-sample data to any degree, but offers no structural or intuitive explanation for the observed non-linear behavior. While providing some structural explanation for the data, the Markov-switching model assumes that the regime switches are exogenous and driven by an unobservable process, and therefore doesn't account for the intuition behind the non-linear policy-behavior.

The STR model on the other hand provides a structural and intuitive framework to explain non-linear policy behavior. In particular, the STR model is a non-linear regression model that allows the regression coefficients to change smoothly from one regime to another - say from a low inflation regime to a high inflation regime. In addition, the STR model allows for endogenous regime switches, and as such provides economic intuition for the non-linear behavior.

If the variables in the Taylor rule are stationary, standard statistical theory applies and the Taylor rule can be estimated in levels. However, if the variables entering the Taylor rule are non-stationary, a co-integrating relationship must exist for the Taylor rule to identify a non-spurious relationship. Hence, it is of utmost importance to know the time series properties of the variables that enter the Taylor rule. This paper uses the Ng-Perron (2001) non-stationarity test, a test with excellent power and size properties.

In particular, the Ng-Perron non-stationarity test is a nearly efficient test, in the sense that it almost achieves the asymptotic power envelope for unit-root tests. For robustness against power issues due to a small sample, the KPSS (1992) stationarity test is used as well. I find that all variables in the Taylor rule are stationary. Thus, standard statistical theory applies, and the STR model is therefore estimated in levels as opposed to in error correction form.

Applying the STR model, I find that the Federal Reserve follows a non-linear Taylor rule during the great moderation period, 1985-2005, and a linear Taylor rule in the dark age of monetary policy, 1960-1979. Therefore, the golden era of monetary policy is associated with the Federal Reserve beginning to follow a time-varying policy-rule. To be sure, once inflation approaches a certain threshold level, the Federal Reserve adjusts its policy-rule and begins to react more strongly to inflation.

Additionally, the fact that the Federal Reserve has switched to a non-linear Taylor rule during the 1985-2005 period, must be held up against a linear Taylor rule satisfying the Taylor principle. Particularly, a Monte-Carlo simulation exercise shows that, if the true data generating process is a non-linear Taylor rule of the STR type, but a linear Taylor rule is estimated, then the coefficient on inflation in the linear Taylor rule will be above 1 - a purely spurious result. Hence, the good monetary policy observed over the last two decades is coupled

with a non-linear Taylor rule and not a linear Taylor rule satisfying the Taylor principle.

The remainder of this paper is organized as follows. Section 2 describes the Taylor rule. Section 3 presents the smooth transition regression model. In section 4 the empirical analysis is performed. Section 5 concludes.

## 2 The Taylor rule

The objectives of monetary policy are price stability (low and stable inflation) and maximum employment (output equal to potential output). The Taylor rule represents a monetary policy strategy for achieving these objectives.

In particular, the Taylor rule is a linear algebraic rule that specifies how the Federal Reserve must adjust its federal funds rate according to the inflation rate and the output-gap,

$$i_t = r + \pi_t + \alpha(\pi_t - \bar{\pi}) + \beta y_t, \quad (1)$$

where  $i_t$  is the nominal federal funds rate,  $r$  is the long-run equilibrium real interest rate,  $\pi_t$  is the year on year inflation rate,  $\bar{\pi}$  is the target inflation rate, and  $y_t$  is the percentage deviation of real GDP from potential output. Taylor (1993) sets the long-run equilibrium real interest rate equal to 2 to match the historical data on real per-capita output growth, the inflation target is set equal to 2, and  $\alpha$  and  $\beta$  are both set equal to 0.5 to allow easy discussion (and because simulation studies performed by Taylor indicated that these values are approximately optimal). Using Taylor's suggested coefficients and rewriting equation (1) yields,

$$i_t = 1 + 1.5\pi_t + 0.5y_t. \quad (2)$$

The coefficient on inflation is constructed to be above one, and illustrates the *Taylor principle*: the Federal Reserve must react more than 1-1 to inflation. This

causes the real interest rate to increase when inflation increases, and has come to represent good monetary policy. The constant represents a linear combination of the long-run equilibrium real interest rate and the inflation target. A change in the intercept is often interpreted as a change in the Federal Reserve's inflation target.

The Taylor rule incorporates many of the features that monetary theory over the past quarter century has identified to be associated with good monetary policy: transparency, accountability and credibility. Especially, a central bank that adheres to a Taylor rule, reveals to the public that it is committed to price stability, and systematically takes steps to achieve it. The public therefore keeps its expectations of inflation low and stable, and financial markets, in addition, anticipate the Federal Reserve's next move and increase market interest rates immediately when inflation picks up.

Taylor (1993) suggests that the Taylor rule should not be followed mechanically, but only in concert with judgment. That is, the Taylor rule corresponds to a guide-post to good monetary policy: a mechanism that constrains monetary policy to be systematic, consistent, and rule-like. Monetary policy that is systematic, consistent, and rule-like characterizes a transparent and credible monetary policy, and therefore alleviates the time-inconsistency problems associated with discretionary monetary policy.

### **3 The Smooth Transition Regression Model**

The smooth transition regression (STR) model is a non-linear time series model. In its most basic form it is equivalent to a linear model with stochastically time-varying coefficients. To be precise, the STR model belongs to the threshold type of non-linear time series models, and as such is capable of explaining threshold behavior. In particular, the STR model allows the regression coefficients to change smoothly from one regime to another, say from a low inflation regime to a high inflation regime. Furthermore, contrary to the Markov-switching model, the STR model allows for endogenous regime switches, and therefore provides economic intuition for the non-linear behavior. Thus, the STR model is capable of explaining why and when the Federal Reserve changes its policy-rule.

The STR model originates with Bacon and Watts (1971) who generalize the Quandt (1958) threshold regression model. Recent accounts include Granger and Teraesvirta (1993), Teraesvirta (1998), Franses and van Dijk (2000) and van Dijk et al. (2002).

The standard STR model is defined as follows,

$$i_t = \phi' \mathbf{z}_t + \theta' \mathbf{z}_t G(\gamma, c, s_t) + u_t, \quad t = 1, \dots, T, \quad (3)$$

and

$$G(\gamma, c, s_t) = (1 + \exp\{-\gamma(s_t - c)\})^{-1}, \quad \gamma > 0,$$

where  $\mathbf{z}_t = (\mathbf{w}_t', \mathbf{x}_t')'$  is a vector of explanatory variables,  $\mathbf{w}_t = (1, y_{t-1}, \dots, y_{t-p})'$  and  $\mathbf{x}_t = (x_{1t}, \dots, x_{kt})'$ , which is a vector of exogenous variables. The parameters  $\phi = (\phi_0, \phi_1, \dots, \phi_m)'$  and  $\theta = (\theta_0, \theta_1, \dots, \theta_m)'$  represent  $((m+1) \times 1)$  parameter vectors in the linear- and non-linear parts of the model, respectively. The disturbance term is *iid* with zero mean and constant variance,  $u_t \sim iid(0, \sigma^2)$ .

The transition function,  $G(\gamma, c, s_t)$ , is bounded,  $G(\gamma, c, s_t) \in [0, 1]$ , and continuous in the threshold variable  $s_t$ . As  $s_t \rightarrow -\infty$ ,  $G(\gamma, c, s_t) \rightarrow 0$  and as  $s_t \rightarrow \infty$ ,  $G(\gamma, c, s_t) \rightarrow 1$ .  $\gamma$  is a slope parameter that determines how smooth the transition between the regimes is, and  $c$  is the threshold around which the different regimes are defined. The threshold variable,  $s_t$ , can be a stochastic variable or a deterministic trend, and can be an element or a linear combination of  $\mathbf{z}_t$ , or a variable not included in  $\mathbf{z}_t$ .

This paper assumes that  $G(\gamma, c, s_t)$  is a logistic function of order one. Hence, equation (3) is more appropriately called a logistic smooth transition regression (LSTR) model. The LSTR model can describe relationships that change according to the level of the threshold variable. For example, if the threshold variable,  $s_t$ , represents the level of inflation, then the LSTR model is able to describe a relationship which properties differ in a high inflation regime from what they are in a low inflation regime. In other words, the LSTR model is capable of explaining asymmetric behavior.



Rewriting equation (3) as,

$$i_t = \{\phi' + \theta' G(\gamma, c, s_t)\} \mathbf{z}_t + u_t, \quad t = 1, \dots, T, \quad (4)$$

provides additional intuition. Equation (4) shows that the STR model is equivalent to a linear model with stochastically time-varying coefficients,

$$i_t = \delta' \mathbf{z}_t + u_t \quad (5)$$

with

$$\delta_j = \phi_j + \theta_j G(\gamma, c, s_t).$$

Given that  $G(\gamma, c, s_t)$  is continuous and bounded between zero and one, each coefficient,  $\delta_j$ , is bounded between  $\phi_j$  and  $\phi_j + \theta_j$ ,  $\delta_j \in [\phi_j, \phi_j + \theta_j]$ , and changes monotonically as a function of  $s_t$ . The closer the threshold variable is to the threshold, and the more it moves beyond the threshold, the closer  $G(\gamma, c, s_t)$  will be to one, and the closer  $\delta_j$  will be to  $\phi_j + \theta_j$ . Similarly, the further the threshold variable falls short of the threshold, the closer  $G(\gamma, c, s_t)$  will be to zero, and the closer  $\delta_j$  will be to  $\phi_j$ .

For example, let  $i_t$  be the federal funds rate, let  $\phi_j = 0.2$  and  $\theta_j = 0.75$  be the linear and non-linear responses to inflation, respectively, and let  $s_t$  be the inflation rate. Then, the Federal Reserve's response to inflation will vary monotonically from 0.2 to 0.95 depending on how close the inflation rate is to the threshold. The closer the inflation rate is to the threshold, and the more it moves beyond the threshold, the stronger the Federal Reserve will respond to inflation.

When  $\gamma = 0$ , the logistic transition function equals 0.5, and the model is linear. That is, the LSTR model nests the linear model. On the other hand, when  $\gamma \rightarrow \infty$ , the LSTR model approaches a threshold regression model with two regimes of equal variances.

## 4 Empirical Analysis

In this section, the variables that enter the Taylor rule are analyzed, and the relationship between them is estimated for the United States.

### 4.1 Data

The data used in this paper are obtained from Fred II and from the BEA. The sample covers the period 1960.1-2005.12.

Several inflation measures exist, but the core personal consumption expenditure index (core PCE) and the core consumer price index (core CPI) are the definitions of inflation that the Federal Reserve follows most closely.<sup>1</sup> Hence, the core PCE and the core CPI are the inflation measures of choice in this paper. The quarterly inflation rates are constructed by taking averages of the monthly inflation time series. The output-gap is constructed by calculating the percentage deviation of real GDP from its HP-trend.<sup>2</sup>

### 4.2 Non-stationarity and stationarity tests

Understanding the time series properties of the variables included in the Taylor rule is critical. If the variables in the Taylor rule are unit-root processes, then a co-integrating relationship must exist for the coefficient estimates to be consistent. Unfortunately, a sizable portion of the literature either does not test for non-stationarity or use unit-root tests with poor size and power properties.

A frequent criticism of unit-root tests concerns the poor size and power properties that these tests have. This is especially true for the Dickey-Fuller (1979) and Phillips-Perron (1988) unit-root tests. However, recent research shows that the Ng-Perron (2001) non-stationarity test has excellent size properties, and a local asymptotic power function that is close to its asymptotic power envelope. See,

---

<sup>1</sup>See, for instance, the bi-annual Humphrey-Hawkins report to the Congress.

<sup>2</sup>To reduce end of sample noise, the HP-filter is applied to a longer sample period, two quarters extra in each end, than the estimated sample. The four extra observations are then excluded from the estimation part.

for instance, Haldrup and Jansson (2006). Hence, this paper uses the Ng-Perron unit-root test.

The considered sample consists of forty years of data, and is thus a small sample, and even the Ng-Perron unit-root test may have low power in such an environment. To hedge against the Ng-Perron unit-root test having low power, the KPSS (1992) stationarity test is used as well.

Table 1: Ng-Perron unit-root test: MZt\*

Variable	$i_t$	$y_t$	$\pi_{1t}$	$\pi_{2t}$
$H_0$ : unit-root	-2.31	-2.28	-1.58	-2.09
Asymptotic critical values: 5%	-1.98	-1.98	-1.98	-1.98
Asymptotic critical values: 1%	-2.58	-2.58	-2.58	-2.58

\* The MZa and MSB tests yield similar results.

$\pi_{1t}$  and  $\pi_{2t}$  represent the core PCE and core CPI inflation measures, respectively.

Table 1 shows that the Ng-Perron non-stationarity test rejects the presence of a unit-root for the federal funds rate, the output-gap, and core CPI inflation, but fails to reject the null for the core PCE inflation index. The fact that the Ng-Perron test is unable to reject the null for core PCE inflation, may be a small sample/power issue. Hence, it is of interest to use the KPSS stationarity test as well, to see whether power is an issue. The KPSS stationarity test results are reported in Table 2.

Table 2: KPSS stationarity test\*

Variable	$i_t$	$y_t$	$\pi_{1t}$	$\pi_{2t}$
$H_0$ : stationarity	0.33	0.02	0.43	0.36
Asymptotic critical values: 5%	0.46	0.46	0.46	0.46
Asymptotic critical values: 1%	0.74	0.74	0.74	0.74

\* Bandwidth 10 (Newey-West with Bartlett kernel).

The KPSS test is unable to reject the null hypothesis of stationarity for each of the variables. Hence, the KPSS test confirms that the federal funds rate, the output-gap, and core CPI inflation are stationary, and further that the core PCE inflation index is hard to classify. In the following, all variables are assumed to be stationary.

### 4.3 Linearity test

Testing for linearity against non-linearity of the threshold type entails testing whether  $\gamma = 0$  in the LSTR model,

$$i_t = \phi' \mathbf{z}_t + \theta' \mathbf{z}_t G(\gamma, c, s_t) + u_t, \quad t = 1, \dots, T, \quad (6)$$

and

$$G(\gamma, c, s_t) = (1 + \exp\{-\gamma(s_t - c)\})^{-1}, \quad \gamma > 0.$$

The LSTR model, however, is not defined under this null, and is only defined under the alternative hypothesis of threshold non-linearity. Fortunately, this identification problem can be circumvented by approximating the transition function with a third order Taylor-series expansion around the null hypothesis

$\gamma = 0$ , see Teraesvirta (1998). The approximation yields, after merging terms and reparameterizing, the following auxiliary regression,

$$i_t = \beta'_0 \mathbf{z}_t + \sum_{j=1}^3 \beta'_j \tilde{\mathbf{z}}_t s_t^j + u_t^*, \quad t = 1, \dots, T, \quad (7)$$

where  $u_t^* = u_t + R_3(\gamma, c, s_t)\theta' \mathbf{z}_t$ , with the remainder  $R_3(\gamma, c, s_t)$ , and  $\mathbf{z}_t = (1, \tilde{\mathbf{z}}_t')'$  where  $\tilde{\mathbf{z}}_t$  is a  $(m \times 1)$  vector of explanatory variables.<sup>3</sup> Furthermore,  $\beta_j = \gamma \tilde{\beta}_j$ , where  $\tilde{\beta}_j$  is a function of  $\theta$  and  $c$ . The null hypothesis of linearity, therefore, becomes  $H_0: \beta_1 = \beta_2 = \beta_3 = 0$ .

Because  $u_t^* = u_t$  under the null hypothesis, an LM-type test is appropriate. The resulting asymptotic distribution is an  $\chi^2$  distribution with  $3m$  degrees of freedom under the null hypothesis.<sup>4</sup> In small to moderate size samples, however, the  $\chi^2$ -statistic can be severely size-distorted, and an  $F$ -statistic is recommended instead, see Teraesvirta (1998). The resulting approximate  $F$ -distribution has  $3m$  and  $T - 4m - 1$  degrees of freedom under the null hypothesis. Because of its desirable small sample properties, this paper uses the  $F$ -statistic.

Before the linearity test can be carried out, a choice of threshold variable has to be made. Letting the inflation rate be the threshold variable, provides the best description of the data and the inflation rate is therefore chosen as the threshold variable. Section 4.4 provides more information on how well the LSTR model fits the data.

Using inflation as the threshold variable and applying the linearity test to the Taylor rule yields the following auxiliary regression,

$$i_t = \beta_{00} + \beta_{01}\pi_t + \beta_{02}y_t + \sum_{j=1}^3 (\beta_{j1}\pi_t + \beta_{j2}y_t)\pi_t^j + u_t^*, \quad (8)$$

with  $H_0: \beta_{11} = \beta_{12} = \beta_{21} = \beta_{22} = \beta_{31} = \beta_{32} = 0$ .

The results of using the linearity test on equation (8), are reported in Table 3.

<sup>3</sup> A minor modification of the auxiliary regression is necessary if  $s_t$  is not part of  $\mathbf{z}_t$ .

<sup>4</sup>  $E(s_t^6 \tilde{\mathbf{z}}_t \tilde{\mathbf{z}}_t')$  must exist for this to be valid.

Table 3: p-values of linearity test

	1960.1-1979.1	1985.1-2005.4
$H_0$ : linear model*	0.1151	0.0056
$H_0$ : linear model**	0.1704	0.0000

\* Inflation is represented by the core PCE inflation rate.

\*\* Inflation is represented by the core CPI inflation rate.

Hence, the data is best described by a linear model over the sample period 1960-1979, and by a non-linear model of the threshold type over the sample period 1985-2005. That is, the Federal Reserve appears to have switched from a linear Taylor rule to a non-linear threshold type Taylor rule.

#### 4.4 Estimating the smooth transition regression model

To estimate the STR model, either non-linear least squares (NLLS) or conditional maximum likelihood (CMLE) can be used. For robustness, this paper uses both methods.

Before the STR model can be estimated, a choice of threshold variable has to be made. Inflation is chosen to be the threshold variable because it provides the best fit with the data. To be precise, the STR model with inflation acting as the threshold variable, yields the lowest Schwarz information criteria (SIC).

Letting inflation be the threshold variable, the LSTR model takes the following form,

$$i_t = \phi_0 + \phi_1 \pi_t + \phi_2 y_t + (\theta_0 + \theta_1 \pi_t + \theta_2 y_t) G(\gamma, c, \pi_t) + u_t, \quad (9)$$

and

$$G(\gamma, c, \pi_t) = (1 + \exp\{-\gamma(\pi_t - c)\})^{-1}, \quad \gamma > 0,$$

where  $i_t$  is the federal funds rate,  $\pi_t$  is the inflation rate,  $y_t$  the percentage deviation of real GDP from potential output,  $\gamma$  is the smoothness parameter, and  $c$  is the threshold around which the regimes are defined.

The best fitting model is found by sequentially eliminating regressors using the SIC measure of fit. Tables 4 and 5 present the best fitting LSTR models.

Table 4: The LSTR model\* \*\*

Parameter	$\phi_0$	$\phi_2$	$\theta_1$	$\gamma$	$c$
<u>NLLS</u>					
Estimate	4.02	1.04	0.75	19.81	3.55
Standard error (HAC)	0.38	0.20	0.09	14.15	0.08
p-value	0.00	0.00	0.00		0.00
<u>CMLE</u>					
Estimate	4.02	1.05	0.75	20.89	3.55
Standard error	0.18	0.15	0.08	26.30	0.08
p-value	0.00	0.00	0.00		0.00
Evaluation criteria: SIC = 0.73, adj $R^2$ = 0.67, $\hat{\sigma}_u$ = 1.27.					

\* Sample period: 1985.1-2005.4

\*\* Inflation is represented by the core PCE inflation rate.

Note: since  $\gamma$  is not defined at zero, the p-values are not reported for  $\gamma$ . In addition,  $\gamma$  has a high standard deviation because relatively few observations are located around the threshold.

The threshold Taylor rule, hence, takes the following algebraic form:

$$i_t = 4.02 + 1.04y_t + [0, 0.75]\pi_t, \quad (10)$$

where the coefficient on inflation varies from 0.00 to 0.75 depending on the level of inflation. Specifically, the coefficient will equal 0.00 if inflation is below 3.30 percent, and 0.75 if inflation takes on a value of 3.80 percent or higher. Notice how the Federal Reserve's policy-rule smoothly transits from the low inflation regime to the high inflation regime. Also, the Taylor principle is never satisfied.

Table 4 and equation (10) make it clear that the Federal Reserve only responds to the core PCE inflation rate when it approaches a threshold of 3.55 percent. In other words, during calm times when inflation is well below its threshold level, the Federal Reserve mainly responds to the output-gap. This is consistent with what Federal Reserve chairman Bernanke has called constrained discretion: the Federal Reserve will respond primarily to the real economy only as long as inflation is low and stable.

Table 5: The LSTR model\* \*\*

Parameter	$\phi_0$	$\theta_1$	$\theta_2$	$\gamma$	$c$
<u>NLLS</u>					
Estimate	1.15	1.26	1.17	20.28	1.92
Standard error (HAC)	0.38	0.08	0.20	8.00	0.05
p-value	0.00	0.00	0.00		0.00
<u>CMLE</u>					
Estimate	1.16	1.26	1.17	21.76	1.93
Standard error	0.32	0.10	0.14	19.52	0.08
p-value	0.00	0.00	0.00		0.00
Evaluation criteria: SIC = 0.36, adj $R^2$ = 0.77, $\hat{\sigma}_u$ = 1.05.					

\* Sample period: 1985.1-2005.4

\*\* Inflation is represented by the core CPI inflation rate.

Note: since  $\gamma$  is not defined at zero, the p-values are not reported for  $\gamma$ . In addition,  $\gamma$  has a high standard deviation because relative few observations are located around the threshold.

In algebraic form:

$$i_t = 1.15 + [0, 1.26]\pi_t + [0, 1.17]y_t, \quad (11)$$

where the coefficient on inflation varies from 0.00 to 1.26 and the coefficient on the output-gap varies from 0.00 to 1.17 depending on the level of inflation. Especially, the coefficient on inflation will equal 0.00 if inflation is below 1.6 percent, and 1.26 if inflation takes on a value of 2.20 percent or higher. Furthermore, the coefficient on the output-gap will equal 0.00 if inflation is below 1.60



percent, and 1.17 if inflation takes on a value of 2.20 percent or higher. Hence, the Taylor principle will be conditionally satisfied. To be precise, the Taylor principle will be satisfied once core CPI inflation moves above 1.98 percent.

Table 5 and equation (11) make it clear that the Federal Reserve only responds to the core CPI inflation rate and the output-gap when the core CPI inflation rate approaches a threshold of 1.92 percent. To be sure, during calm times when inflation is well below its threshold level, the Federal Reserve mainly observes the economy and lets it run its course.

Now, changing the inflation index produces different results, but the main message is the same: the Federal Reserve follows a non-linear Taylor rule. Furthermore, the Federal Reserve doesn't begin to state inflation in terms of PCE prior to February 2000, and core PCE in July 2004, in its monetary policy report to the Congress. Thus, the results derived using the CPI inflation measure probably best represent Federal Reserve behavior during the period of interest.

Given that the Federal Reserve follows a non-linear Taylor rule, the Lucas (1976) critique becomes very relevant: with the Federal Reserve following a time-varying policy-rule, the decision rules of private agents must be expected to change over the business cycle as well.

## 4.5 The linear Taylor rule vs. the non-linear Taylor rule

To conclude that the Federal Reserve follows a non-linear Taylor rule only is a valid claim if the non-linear Taylor rule provides a better fit with the data than the linear Taylor rule does. This section compares the two policy-rules along several dimensions.

Before the two models can be compared, the linear Taylor rule has to be estimated. Recall, the linear Taylor rule takes the form,

$$i_t = \phi_0 + \phi_1 \pi_t + \phi_2 y_t + u_t.$$

Tables 6 and 7 report the OLS estimates of the linear Taylor rule.

Table 6: The linear Taylor rule\* \*\*

Parameter	$\phi_0$	$\phi_1$	$\phi_2$
Estimate	1.90	1.19	1.15
Standard error (HAC)	0.74	0.20	0.19
p-value	0.01	0.00	0.00

Evaluation criteria: SIC = 0.77, adj  $R^2$  = 0.63,  $\hat{\sigma}_u$  = 1.37.

\* Sample period: 1985.1-2005.4

\*\* Inflation is represented by the core PCE inflation rate.

Table 7: The linear Taylor rule\* \*\*

Parameter	$\phi_0$	$\phi_1$	$\phi_2$
Estimate	0.62	1.40	1.14
Standard error (HAC)	0.63	0.15	0.16
p-value	0.32	0.00	0.00

Evaluation criteria: SIC = 0.38, adj  $R^2$  = 0.75,  $\hat{\sigma}_u$  = 1.13.

\* Sample period: 1985.1-2005.4

\*\* Inflation is represented by the core CPI inflation rate.

Comparing the two models in terms of the SIC, adjusted  $R^2$ , and standard error of the residual, provides valuable information about the fit of each model.

Table 8: The linear vs. non-linear Taylor rule\*

Criteria	Linear(a)	Non-linear	Linear(b)	Non-linear
SIC	0.77	0.73	0.38	0.36
Adj $R^2$	0.63	0.67	0.75	0.77
Standard error	1.37	1.27	1.13	1.05

\* Sample period: 1985.1-2005.4

a: inflation is represented by the core PCE inflation rate.

b: inflation is represented by the core CPI inflation rate.

Table 8 shows that the non-linear Taylor rule improves upon the linear model, along all evaluation dimensions. Hence, the non-linear Taylor rule provides a better description of monetary policy over the last two decades.

#### 4.6 The Taylor principle vs. a non-linear Taylor rule

If the true policy-rule is non-linear, but a linear Taylor rule is estimated, then the estimated linear Taylor rule represents a linear approximation to the true policy-rule. To be precise, the estimated linear Taylor rule is meaningless: the misspecified functional form leads to spurious coefficient estimates.

Table 9 presents a Monte-Carlo simulation study on how sensitive the coefficients in the linear Taylor rule are to misspecification of the functional form. In particular, if the true Taylor rule is non-linear of the following LSTR type,

Case A:

$$i_t = 4.02 + 1.05y_t + (0.75\pi_t)G(\gamma, c, \pi_t) \quad (12)$$

and

$$G(\gamma, c, \pi_t) = (1 + \exp\{-20(\pi_t - 3.55)\})^{-1},$$

Case B:

$$i_t = 1.15 + (1.26\pi_t + 1.17y_t)G(\gamma, c, \pi_t) \quad (13)$$

and

$$G(\gamma, c, \pi_t) = (1 + \exp\{-20(\pi_t - 1.92)\})^{-1},$$

but a linear Taylor rule is estimated,

$$i_t = \phi_0 + \phi_1 \pi_t + \phi_2 y_t + u_t,$$

then the following results hold:<sup>5</sup>

Table 9: The Taylor principle\* \*\*

Parameter	$\phi_0$	$\phi_1$	$\phi_2$
Estimate(a)	0.81	1.37	1.05
t-statistic	1.75	10.67	7.15
Estimate(b)	-0.09	1.58	1.02
t-statistic	-0.21	13.34	7.60

\* 10000 simulations of samples of size 84.

\*\* The results are robust to the choice of  $\gamma$  for  $\gamma$  above 3.

a: Case A model used to generate the interest rate data.

b: Case B model used to generate the interest rate data.

The simulation exercise shows that, if the true policy-rule is non-linear but a linear Taylor rule is estimated, then the researcher is misled to conclude that the Federal Reserve satisfies the Taylor principle.

Hence, the fact that the coefficient on inflation is above 1 in a linear Taylor rule has misled economists, see, for instance, Clarida et al. (1999, 2000), to conclude that the stellar monetary policy observed throughout the past two decades is associated with the Federal Reserve following a linear Taylor rule that satisfies the Taylor principle.

## 5 Conclusion

While there is a vast amount of literature available dealing with the linear Taylor rule, little is known about non-linear Taylor rules. This paper is an attempt at

<sup>5</sup>Inflation is modeled to have an equal number of observations above and below the threshold, but otherwise has the same standard deviation as observed inflation. The output-gap is modeled to have the same mean and standard deviation as the observed output-gap.

estimating a non-linear Taylor rule.

The smooth transition regression model provides a natural framework to estimate a non-linear Taylor rule. The STR model allows for regime changes to happen endogenously, and is therefore capable of explaining why and when the Federal Reserve changes its policy-rule.

Estimating the STR model shows that a non-linear Taylor rule fits the data better than a linear Taylor rule during the great moderation, 1985-2005, but that a linear Taylor rule better describes the data during the great inflation, 1960-1979. Thus, the golden era of monetary policy is associated with the Federal Reserve adopting a non-linear Taylor rule. In particular, when inflation approaches a certain threshold, the Federal Reserve adjusts its policy-rule and begins to react more forcefully to inflation (and the output-gap).

What is more, the good monetary policy observed during the last two decades, is related to a non-linear Taylor rule that never or only conditionally satisfies the Taylor principle. Thus, a Taylor rule that only conditionally satisfies the Taylor principle is sufficient for good monetary policy.

## References

- [1] Bacon, D.W., Watts, D.G. (1971). 'Estimating the transition between two intersecting straight lines', *Biometrika*, vol. 58, pp. 525-34.
- [2] Beaudry, P. and Koop, G. (1993). 'Do recessions permanently change output?', *Journal of Monetary Economics*, vol. 31, pp. 149-64.
- [3] Clarida, R., Gali, J. and Gertler, M. (1999). 'The science of monetary policy: A new Keynesian perspective', *Journal of Economic Literature*, vol. 37, pp. 1661-1707.
- [4] Clarida, R., Gali, J. and Gertler, M. (2000). 'Monetary policy rules and macroeconomic stability: evidence and some theory', *Quarterly Journal of Economics*, vol. 115, pp. 147-180.
- [5] Dickey, D.A. and Fuller, W.A. (1979). 'Distribution of the estimators for autoregressive time series with a unit root', *Journal of the American Statistical Association*, vol. 74, pp. 427-431.

- [6] Dolado, J.J., Maria-Dolores, R. and Naveira, M. (2005). 'Are monetary policy reaction functions asymmetric?: the role of nonlinearity in the Phillips curve', *European Economic Review*, vol. 49, pp. 485-503.
- [7] Franses, P.H. and van Dijk, D. (2000). 'Non-linear time series models in empirical finance', *Cambridge University Press*, Cambridge.
- [8] Granger, C. and Teraesvirta, T. (1993). 'Modeling nonlinear economic relationships', *Oxford University Press*, Oxford.
- [9] Haldrup, N. and Jansson, M. (2006). 'Improving power and size in unit root testing', *Palgrave Handbooks of Econometrics*, vol. 1 Econometric Theory, chapter 7, Palgrave MacMillan, Basingstone.
- [10] Hamilton, J.D. (1989). 'A new approach to the economic analysis of non-stationary time series and the business cycle', *Econometrica*, vol. 57, pp. 357-84.
- [11] Kwiatkowski, D., Phillips, P.C.B., Schmidt, P. and Shin, Y. (1992). 'Testing the null hypothesis of stationarity against the alternative of a unit root', *Journal of Econometrics*, vol. 54, pp. 159-78.
- [12] Lucas, R. (1976). 'Econometric policy evaluation: A critique', *Carnegie-Rochester Series on Public Policy*, I, pp. 19-46.
- [13] Neftci, S.N. (2001). 'Are economic time series asymmetric over the business cycle?', *Journal of Political Economy*, vol. 92, pp. 307-28.
- [14] Ng, S. and Perron, P. (2001). 'Lag length selection and the construction of unit root tests with good size and power', *Econometrica*, vol. 69, pp. 1519-54.
- [15] Nobay, A.R. and Peel, D. (2003). 'Optimal discretionary monetary policy in a model of asymmetric central bank preferences', *Economic Journal*, vol. 113, pp. 657-65.
- [16] Phillips, P.C.B. and Perron, P (1988). 'Testing for a unit root in time series regression', *Biometrika*, vol. 75, pp. 335-46.
- [17] Quandt, R.E. (1958). 'The estimation of parameters of a linear regression system obeying two separate regimes', *Journal of the American Statistical Association*, vol. 53, pp. 873-80.

- [18] Ruge-Murcia, F.J. (2003). 'Inflation targeting under asymmetric preferences', *Journal of Money, Credit and Banking*, vol. 35, pp. 763-85.
- [19] Surico, P. (2006). 'The Fed's monetary policy rule and U.S. inflation: the case of asymmetric preferences', *Journal of Economic Dynamics and Control*, forthcoming.
- [20] Svensson, L.E.O. (1997). 'Inflation forecast targeting: implementing and monitoring inflation targets', *European Economic Review*, vol. 41, pp. 1111-46.
- [21] Taylor, J. (1993). 'Discretion versus policy rules in practice', *Carnegie-Rochester Series on Public Policy*, XXXIX, pp. 195-214.
- [22] Teraesvirta, T. (1998). 'Modeling economic relationships with smooth transition regressions', in Ullah, A. and Giles, D.E. (eds.), *Handbook of Applied Economic Statistics*, Dekker, New York, pp. 507-52.
- [23] van Dijk, D. and Franses, P.H. (1999). 'Modeling multiple regimes in the business cycle', *Macroeconomic Dynamics*, vol. 3, pp. 311-40.
- [24] van Dijk, D., Teraesvirta, T. and Franses, P.H. (2002). 'Smooth transition autoregression models - A survey of recent developments', *Econometric Reviews*, vol. 21, pp. 1-47.