May 2008

The Race to the Suburb: The Location of the Poor in a Metropolitan Area

Charles A. M. de Bartolome
University of Colorado

Stephen L. Ross
University of Connecticut

University of Connecticut

Department of Economics Working Paper Series

The Race to the Suburb: The Location of the Poor in a Metropolitan Area

Charles A. M. de Bartolome
University of Colorado

Stephen L. Ross
University of Connecticut

Working Paper 2002-02R

August 2002, revised May 2008

---

341 Mansfield Road, Unit 1063
Storrs, CT 06269–1063
Phone: (860) 486–3022
Fax: (860) 486–4463
http://www.econ.uconn.edu/

Abstract

We provide an explanation for the stylized fact that poor households are concentrated in the inner city of most U.S. metropolitan areas. We consider a metropolitan area with an inner city surrounded by a suburb and two income classes. Using numerical simulations, we show that two equilibria typically exist: one in which the inner city has a majority of poor households and the other in which it has a majority of rich households. We argue that the growth path selects the former equilibrium because rich households jump to the suburb before poor households spill into the suburb. In addition, the model provides an explanation for gentrification: at large metropolitan populations, population growth causes rich households in the city to live in areas previously inhabited by poor households.

Journal of Economic Literature Classification: H73, R12, R14

Keywords: urban, equilibria, poor

1. INTRODUCTION

In most U.S. metropolitan areas poor households are concentrated in the inner city. For example, Glaeser et al. (2000) report "the well-documented fact that within U.S. metropolitan areas, the poor generally live in the central cities and middle-income households generally live in the suburbs." At first glance this is somewhat surprising as the concentration of jobs in the inner city and the higher cost of commuting time for rich households might be expected to lead rich households to outbid poor households for the locations closer to the metropolitan center. This paper uses the growth path of the metropolitan area to explain the paradox. In addition it provides an explanation for gentrification in the inner city.

The "monocentric city" model of Alonso (1964) Mills (1967) and Muth (1961, 1969) adds land demand to the simple model of commuting cost in an early attempt to explain the concentration of the rich in the suburbs (leaving the poor in the inner city). All jobs are considered to be located at the metropolitan center. When deciding where to live, a household is considered to trade-off commuting costs and land prices. If a household buys a house at a location closer to the metropolitan center, it spends less time commuting and this advantage is capitalized by a rise in the land price. Because rich households buy more land, the low price of land in the suburbs pulls them towards the suburbs. Whether this is sufficient to overcome the higher commuting cost depends on the magnitude of the two effects. If land demand is more income elastic than commuting costs the lower land price in the suburbs dominates and rich households are predicted to concentrate in the suburbs. Conversely, if land demand is less income elastic than commuting cost, commuting considerations dominate and rich households are predicted to concentrate in the inner city (Wheaton (1977)).

Estimates of the income elasticity of land demand do not support the equilibrium with rich households being concentrated in the suburbs. Wheaton (1977) estimates that the income elasticity of land demand is statistically indistinguishable from the income elasticity of the commuting cost, so that the Alonso-Mills-Muth model is unable to predict whether it is poor households or rich households who live in the inner city. Glaeser, Kahn and Rappaport (2000) find evidence that the income elasticity of land demand is significantly less than the income elasticity of commuting, so that the monocentric city model predicts that it is the rich households who live in the inner city. Empirically, therefore, this type of sorting on its own cannot be an explanation for the centralization of the poor.

Tiebout's (1956) model of fiscal decentralization stresses that public services are important determinants of where households reside. A household shops over jurisdictions, choosing the jurisdiction which provides his preferred public service level. Tiebout's model is normative as he seeks to establish the efficiency of the resulting equilibrium. However, many authors (e.g., Elickson (1971), McGuire (1974), Berglas (1976a, 1976b), Wooders (1978), Yinger (1982) and Epple et al. (1984, 1993)) have extended the model to consider positive outcomes. These authors show that, because households with different incomes have different demands for the public service, they choose different jurisdictions, or there is sorting by income between jurisdictions. Differences in public service levels are capitalized into land prices.[1]

Tiebout's model is non-spatial. Jurisdictions are viewed as areas of land in a featureless plain so that there is no a-priori reason as to which jurisdiction or which piece of land is inhabited by the poor households. If there are two jurisdictions, labeled the inner city and the suburb, then there are two equilibria: one where the inner city contains the poor households, and another where the suburb contains the poor households. Tiebout's model therefore suggests that households do sort by income between jurisdictions but provides no prediction as to whether the poor households congregate in the inner city or the suburb.

Our view of the topic (as discussed in de Bartolome and Ross (2003, 2004, 2007)) is that commuting costs, land prices and public service levels are all important determinants as to where households locate. In our model, a circular inner city has an exogenous boundary and is surrounded by a suburb. Commuting considerations are present because all households must commute to the central business district which is located at the center of the inner city. In addition, households care about the public service provided by a jurisdiction and its level is determined by voting. The model has two income-classes. Rich households have higher commuting costs per mile than poor households and, consistent with the data, land demand is relatively income inelastic. *Ceteris paribus*, therefore, rich households outbid poor households for land nearer the inner-city's center. In addition, rich households have a higher demand for the public service so that *ceteris paribus* different income groups prefer to live in different jurisdictions. In the spirit of the indeterminacy of Teibout's model, we find two equilibria over a range of metropolitan populations. In one equilibrium, it is the poor households who form the majority in the inner city, voting low public services in that city; in the second equilibrium, it is the rich households who form the majority in the inner city, voting high public services there. What is unexplained in our earlier work is why the equilibrium with poor households forming the majority in the inner city has been selected by most U.S. metropolitan areas. This is the topic addressed by this paper.

To determine which equilibria is likely to be selected, we simulate the city's growth by considering an increase in the metropolitan population in the presence of a fixed boundary between the inner city and the suburb. Poor households are the majority in the metropolitan population. When the population is small, the equilibrium has all households living in the inner city; poor households, forming the majority, vote a low level of the public service. As the population increases, the edge of urban development moves outwards towards the inner-city's boundary and city rents increase . While there is still some undeveloped land in the inner city, some rich households "jump" to the suburb to form a new jurisdiction with a high public service. This establishes rich households as the majority in the suburb. Further growth in the metropolitan population leaves this configuration in place: rich households congregate in the suburb leaving poor households in the inner city.

Other authors have provided possible explanations as to why the poor are concentrated in the inner cities. LeRoy and Sonstelie (1983) suggest that it may be a consequence of the introduction of the automobile. In their model, car travel is faster than public transportation but is also more expensive - initially therefore cars were bought by rich households allowing them to move out of the inner city. Glaeser et al. (2000) suggest that the reason lies in public transportation. In their model, public transportation is favored by high population density and is therefore located in the inner city. Poor households use public transportation to commute and hence they locate in the inner city where the public transportation is. Brueckner and Rosenthal (2006) suggest that the reason lies in the housing stock: richer households live in the suburbs because they are attracted by the newer housing stock there.

Although "the poor generally live in the cental cities and middle-income households generally live in the suburbs" (op. cit.), sorting is incomplete. As is well-known, inner cities contain many middle-income households and the suburbs contain many poor households. In our earlier papers we showed how capitalization supports an equilibrium in which both income classes live in both jurisdictions ("income-mixing"). In this paper we show that, in this equilibrium configuration, population growth causes the boundary between the rich and poor households in the inner city to move outwards so that areas which were previously inhabited by poor households become inhabited by rich households - a process which is descriptively similar to the "gentrification" observed in many U.S. cities since the 1990s.

The purposes of this paper are essentially positive. We therefore want a model which is simple enough to show the underlying forces and yet rich enough to capture the important institutional details.[2] Our main simulation has local government financing itself using both a property tax levied on homes and a residence tax, where the latter is used to represent non-residential sources of revenue (viz. the property tax levied on business, the sales tax and intergovernmental grants). However, our main focus is on establishing the growth path of the metropolitan area in the presence of commuting forces, land demand and public service differences and the property tax - by shifting the tax burden from poor households to rich households - affects the incentives of households when choosing where to locate. Therefore, to establish that our results are not due to the property tax *per se*, we rerun the simulations with the residence tax being the only source of local revenue. The movement of rich households to the suburb is maintained.

Because we are comparing different equilibria, it is difficult to use a calculus-based methodology. We therefore use a computable general equilibrium model. We also use a very simple utility function so that the intuition is highlighted. The paper is structured as follows:

Sections 2 and 3 present the theoretical model; Section 4 presents the simulation structure; Section 5 discusses the calculated equilibria, the selection of the equilibrium and gentrification; and Section 6 concludes.

2. THE MODEL

2.1 Spatial overview

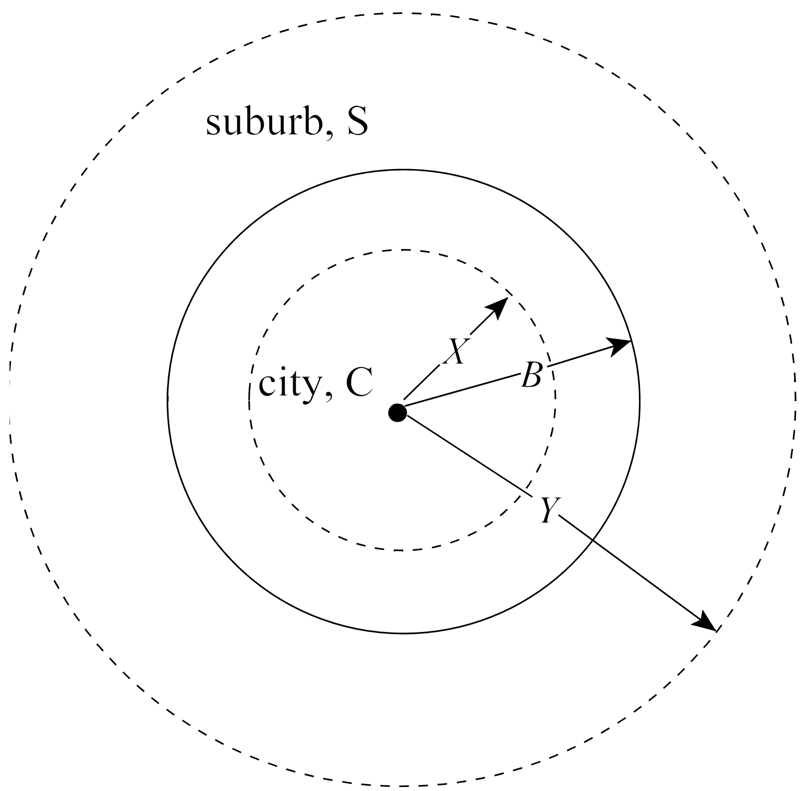


Figure 1: the metropolitan area

The spatial layout of the metropolitan area is illustrated by Figure 1. At the center of the metropolitan area is the business district to which all households must commute; for ease of presentation the business district is assumed to be a point with no area. The business district is surrounded by a circular central city, henceforth denoted as "the city"and labeled *C*. The city has an exogenous jurisdictional boundary of radius B. In the city there may be undeveloped land, so that the limit of development has radius X:

$X < B$: there is undeveloped land at the edge of the city;

$X = B$: there is no undeveloped land in the city.

The city is surrounded by a suburb, labeled S. The outer jurisdictional boundary of the suburb is sufficiently distant that all households live in the city or in the suburb; the outer limit of development in the suburb is a circle of radius Y. Our interest is in how households of differing incomes distribute themselves across the metropolitan area as the metropolitan population grows.

2.2 Basic Analytic Structure

An household lives in a jurisdiction j ($j \in \{C, S\}$) and obtains utility from consuming c units of a privately-provided numeraire good, from consuming l units of land, from consuming h units of housing capital and from g^j units of a public service provided by the jurisdiction. For ease of calculation, we consider a utility function which is linear in the numeraire and additively separable in its arguments:

$$U = c + \alpha u(l) + \beta v(h) + \gamma w(g^j),$$

where $u(.)$, $v(.)$ and $w(.)$ are strictly concave functions. Because we want the demand for land, housing capital and the public service to appear normal or to be more valued by households of higher income, we set α, β and γ to be functions of the endowed income of the household , M :

$$\alpha \equiv \alpha(M) \ ; \ \beta \equiv \beta(M) \ ; \ \gamma \equiv \gamma(M) \ ; \ \partial\alpha/\partial M > 0 \ ; \ \partial\beta/\partial M > 0 \ ; \ \partial\gamma/\partial M > 0 \ .$$

In this description, households differ in their tastes for land, housing capital and the public service, and their tastes vary systematically with endowed income.[3]

Each household has a fixed time endowment which he can use either for working or for commuting to the metropolitan center. His endowed income M is his income if he spends no time commuting or if he lives at the metropolitan center. If he lives at distance z from the metropolitan center, his income is reduced by the opportunity cost of the commute. The time spent commuting is proportional to z and the opportunity cost of a unit of his time is proportional to M, so that in this case his commuting cost is $kM z$ (where k is a constant). There is the possibility of a lump-sum transfer T to all households. Hence his income available to buy the numeraire good, to buy land and housing capital and to pay taxes is $M - kMz + T$.

The jurisdiction provides the public service g^j . The production of the public service shows constant returns to jurisdiction size, and the cost of providing a unit of the public service to a resident is s (units of numeraire per resident)[4] . Most U.S. local governments finance public services using a property tax levied on homes, a property tax levied on businesses, a sales tax and intergovernmental grants. To model this "mixed" revenue structure by each jurisdiction financing a fraction λ of its cost of the public service using a property tax on land and housing capital levied at tax rate t^j, and financing the remaining fraction $1 - \lambda$ using a residence tax. In our main simulation we consider $\lambda = .4$ to be a good approximation to current U.S. practice. However, our focus is on sorting between jurisdictions based on differences in commuting cost, land demand and public services, and the use of the property tax introduces additional incentives (relative to the residence tax). First: rich households spend more on their homes than poor households so that, in jurisdictions in which both income classes reside, rich households pay a greater tax share or there is an implicit transfer from rich to poor households. This makes such jurisdictions less attractive to rich households and more attractive to poor households. Second: in jurisdictions in which both income classes reside, the property tax lowers the tax-price of the public service to poor households - leading them to vote a higher public service level if they are the majority. This makes such jurisdictions more attractive to rich households. To show that our results are driven by the difference in public service levels and not by the property tax *per se*, we consider the case of the pure residence tax ($\lambda = 0$) after presenting the main simulations.

The price of a unit of land at distance z from the metropolitan center is $r(z)$ and the price of a unit of housing capital is p. The consumption of the private good by the household if he locates at distance z from the metropolitan center is therefore

$$c = M - kMz + T - (1-\lambda)sg^j - r(z)(1+t^j)l - p(1+t^j)h \; ;$$

the utility of the household is

$$M - kMz + T - (1-\lambda)sg^j - r(z)(1+t^j)l - p(1+t^j)h + \alpha(M)u(l) + \beta(M)v(h) + \gamma(M)w(g^j).$$

Budget balance by the jurisdiction requires that the tax rate is set as

$$t^j \, \Sigma_{\text{all households resident in jurisdiction } j} \, (rl + ph) = (\text{number of residents in jurisdiction } j)\, \lambda s g^j .$$

There are two income classes. Poor households have income M_1 and rich households have income M_2: $M_1 < M_2$.

2.3 Rents and sorting within a jurisdiction

At equilibrium, a household of income M achieves utility $U(M)$. Denote the bid of a household of income M for land at a location which is distance z from the metropolitan center and lies in jurisdiction j as $\mathrm{R}(z\,;\,j\,,M)$:

$$\max_{l,h} M - kMz + T - (1-\lambda)sg^{j} - R(z;j,M)(1+t^{j})l - p(1+t^{j})h$$

$$+ \alpha(M)u(l) + \beta(M)v(h) + \gamma(M)w(g^{j}) = U(M)\,.$$

Differentiating with respect to z within a jurisdiction, using the envelope condition, and rearranging:

$$(1+t^{j})\,\frac{\partial R(z\,;j,M)}{\partial z} = -\frac{kM}{l}\,. \tag{1}$$

The willingness-to-pay (per unit of land) of the household to move marginally closer to the metropolitan center is the benefit of the decreased commuting cost per unit of land area purchased.

Differentiating with respect to M and rearranging:

$$\frac{\partial}{\partial M}\frac{\partial R}{\partial z} = -\frac{k}{(1+t^{j})l}\left(1-\frac{M}{l}\frac{\partial l}{\partial M}\right).$$

The bid-rent curve steepens (becomes more negative) with income if the income elasticity of land demand is less than unity.[5] Recent empirical estimates suggest that the income elasticity of land demand is less than unity, and this case is henceforth assumed.

ASSUMPTION: the income elasticity of land demand is less than unity: $l/M \, \partial l / \partial M < 1$.

The rent paid at any location is the highest bid-rent of all households at that location or the rent schedule *r(z)* in the jurisdiction is the envelope of the bid-rent functions. A household locates at the point where his bid-rent curve touches the envelope, or

$$\frac{\partial R(z\,; j\,, M)}{\partial z} = \frac{dr(z)}{dz}. \tag{2}$$

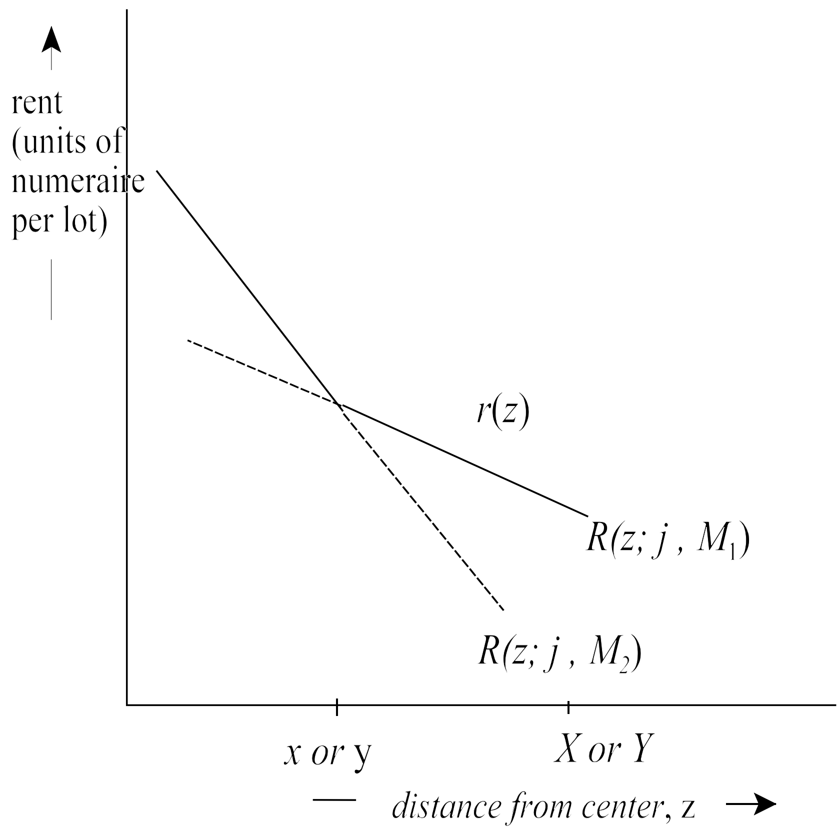


Figure 2: the rent schedule as the envelope of the bid-rent curves

Figure 2 illustrates the creation of the rent schedule in a jurisdiction if both income classes reside there. It shows the bid-rent curves $R(z\ ; j, M_1)$ and $R(z; j, M_2)$, with the bid-rent curve of the rich household being steeper than the bid-rent curve of the poor household. The rent schedule $r(z)$ is the envelope of the bid-rent curves.[6] Rich households are outbidding the poor households for the locations closer to the metropolitan center because the benefit to them of the saved commuting is greater. Households of income M_2 locate on the inside of the jurisdiction and households of income M_1 locate on the outside of the jurisdiction, or income decreases as distance from the metropolitan center increases. With both income classes living in the city (suburb), the boundary between the income classes in the city (suburb) occurs at distance x (y) from the metropolitan center.[7]

The rent schedule in a jurisdiction must be continuous as otherwise a household who is located adjacent to the discontinuity on the side of high rent could increase his utility by moving across the discontinuity to the side of low rent: his rent would decrease by a discrete amount but his commuting cost would increase only marginally.

These results are summarized in the Lemma:

LEMMA : *Within a jurisdiction:*

(a) rent decreases as distance from the metropolitan center increases.

(b) if both income classes reside in a jurisdiction, rich households live on the inside and poor households live on the outside.

(c) the rent schedule is continuous.

2.4 Sorting between jurisdictions

A household chooses to live in the jurisdiction in which he achieves the greatest utility. If households of the same income live in both jurisdictions, they must achieve the same utility in each jurisdiction: any commuting or fiscal gain the households achieve in one jurisdiction is exactly balanced by the higher rent they have to pay. If both jurisdictions are inhabited but no households of income M live in one jurisdiction, then rents in that jurisdiction are such that a household of income M cannot increase his utility by moving into that jurisdiction. If a jurisdiction is uninhabited, the household calculates the utility he would achieve by moving into the jurisdiction by assuming that the rent he would pay is the reservation rent of land (see below) and that, by moving, he would become the majority so that the public service level he would experience would be his desired public service level.[8]

2.5 Model closure

The public service in each jurisdiction is set by majority voting; households vote myopically, taking the rent schedule and the jurisdictional population as given.[9]

In our simulations, there is an equal number of poor and rich households. This has the advantage of ensuring that, if both jurisdictions are occupied, one has a majority of poor households and one has a majority of rich households. The division of the population into two income classes is of course artificial and, understanding that the U.S. income distribution is skewed towards poor households, we assume that, in the case of tied voting (a situation which arises when only the city is inhabited), the voted outcome is the outcome desired by poor households.

The model is closed by assuming:[10]

1. The number of households in the metropolitan population, N, is exogenous. N is considered to be a continuous variable.
2. The reservation price of land is r_0. The rent at the limit of development in the suburb (Y) is therefore r_0 . If the city contains undeveloped land ($X < B$), the rent at the limit of development (X) is r_0. If all the city's land is developed ($X = B$), the rent at the city's side of the jurisdictional boundary is at least r_0.
3. The average rent paid by all households is returned to households as the lump-sum transfer T. [11]

3. GROWTH: SUBURBANIZATION AND GENTRIFICATION

We now consider the growth of the metropolitan population in order to explain why poor households tend to be concentrated in the city and gentrification. Our presumptions are:

(1) Equilibrium configurations. As the metropolitan area grows, the equilibrium configuration at each point in time resembles a static equilibrium for the contemporaneous population level.

(2) Continuity. As the metropolitan population grows from N to $N + \Delta N$, the equilibrium configuration changes from one static equilibrium to another static equilibrium. The new static equilibrium at population $N + \Delta N$ is one which can be reached from the pre-existing equilibrium (associated with population N) by marginal changes in the limits of development X and Y and in the within-jurisdiction boundaries between the

income classes x and y (if such an equilibrium exists). Only if there is no such "adjacent" equilibrium is there large-scale migration between the jurisdictions.

Historical metropolitan populations were very small. All households lived in the city and the city had undeveloped land ($X < B$) . We describe this equilibrium configuration as the "single city". Poor households are the "as if" majority and vote a low public service. If a rich household moved to the suburb, he would benefit from being able to vote a high level of the public service and the lower rent, but he is deterred from doing this by the high cost of commuting from the suburb.

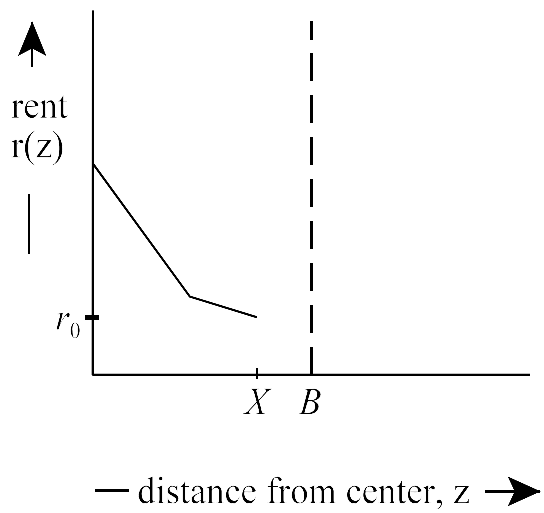


Figure 3: equilibrium configuration of the "single city"

In the remainder of this paper, we illustrate an equilibrium configuration by using its rent schedule. Figure 3 illustrates the "single city" using its rent schedule. Our assumption that the income elasticity of land demand is less than unity implies that the rent gradient is steeper in the region inhabited by rich households. Therefore, in interpreting the figures, we can use the slope of the rent schedule to infer the income of a household living at a location: a flat (steep) rent schedule implies that poor (rich) households live at that location.[12] For example, interpreting Figure 3: moving in from the jurisdictional boundary, initially there is an undeveloped region; at the limit of development (*X*) the rent is the reservation value r_0 ; then the rent schedule is relatively flat indicating that these locations are inhabited by poor households; at the class boundary *x* the rent schedule steepens indicating that the locations, which are closer to the metropolitan center than *x*, are inhabited by rich households.

As the metropolitan population grew, the limit of development in the city moved outwards and eventually such growth led to the development of a suburb. One income class became the majority in the suburb, leaving the other income class to become the majority in the city. Presumption (2) implies that whichever income class became first established in the suburb retained its majority in the suburb, leaving the other income class to retain its majority in the city.[13] Thus, to predict which equilibrium configuration is selected - with poor or rich households forming the majority in the city - we need to determine which income class enters the suburb first. This is the "race to the suburb" referred to in the title of this paper. There are two scenarios:

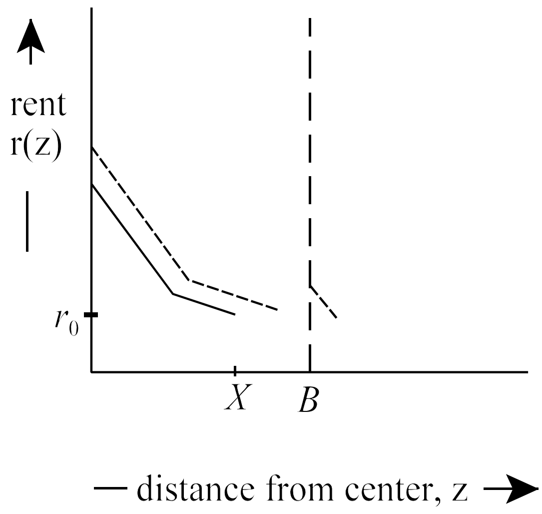


Figure 3: the rich "jump" over undeveloped land to the suburb

1. *Rich households enter the suburb first by "jumping" over undeveloped land.* Starting from the configuration of the "single city", as the population grows rents rise at the center. At a critical population and while there is still undeveloped land in the city, a rich household may find it beneficial to move to the suburb. In the suburb, he would vote his desired public service level and pay a lower rent. Although he incurs a higher commuting cost, he will benefit overall if he is sufficiently sensitive to the public service level. The property tax provides an additional benefit because, by moving to the suburb, he avoids paying the implicit transfer to the poor. The metropolitan growth in this scenario is illustrated in Figure 3: the solid line shows the equilibrium configuration at the pre-existing metropolitan population and the dashed lines show the new equilibrium configuration which has some rich households inhabiting the suburb. With rich households leaving the city, poor households are left in control of

the city. We term this scenario "jumping" over undeveloped city land.[14]

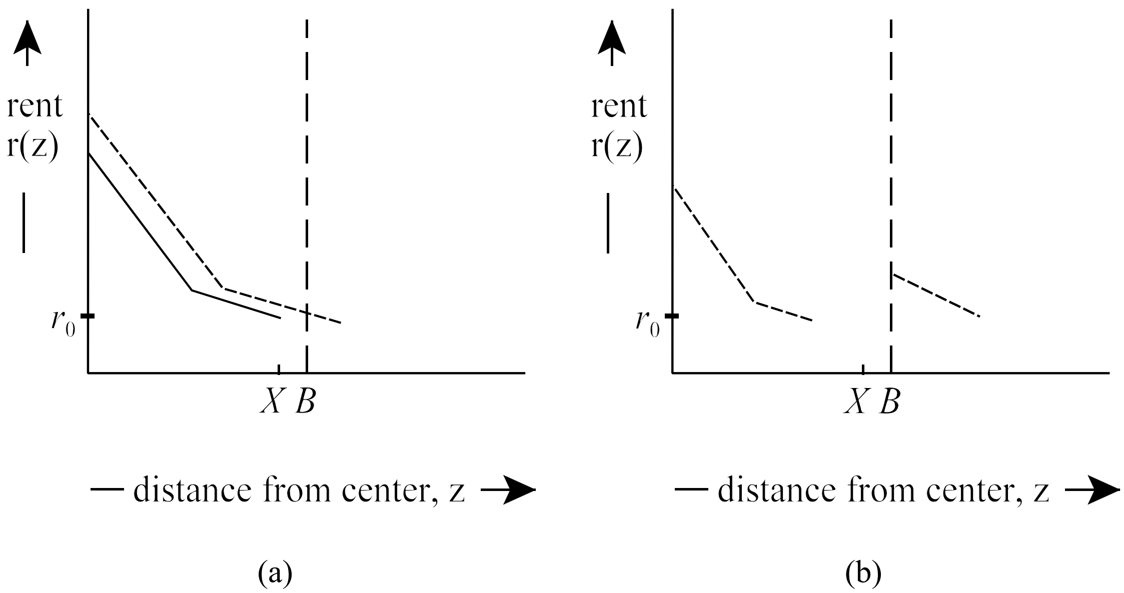


Figure 4: poor households "spill" into the suburb

2. *Poor households enter the suburb first by "spilling" from the city*. Unlike rich households, poor households have no incentive to set up a suburban jurisdiction while the "single city" still has undeveloped land. To see this, consider a "single city" represented by the solid line in Figure 4(a) and consider a poor household at the edge of development in the city. If he were to move to the suburb, he would pay the same rent, the public service would still be his desired level but he would incur a higher commuting cost. The property tax provides an additional cost because, by moving to the suburb, he would pay higher taxes (the implicit transfer from the rich is forfeited). Therefore in this scenario poor households can enter the suburb first only by "spilling over" into the suburb when the limit of city development moves across

the jurisdictional boundary. The dashed line in Figure 4(a) illustrates the new instantaneous configuration which now has some poor households inhabiting the suburb.

In this scenario, as poor households "spill" into the suburb, rich households become the city's majority and they vote a high public service level.[15] The high public service level makes the city less attractive to poor households and some additional poor households leave the city for the suburb, recreating undeveloped land in the city. This would cause a discontinuous change in *x, X* and *Y*. If the poor maintained its majority in the suburb, the equilibrium configuration would resemble that illustrated in Figure 4(b). However, if there are several possible equilibria, continuity alone is unable to predict which equilibrium is chosen.[16]

The critical distinction between the two scenarios is whether the rich become established in the suburb while there is still undeveloped land in the city. To establish which scenario is realized, we simulate the metropolitan area at different population levels.

In addition to providing an explanation as to why poor households concentrate in the city, our simulation also provides an explanation for gentrification. Gentrification occurs in the city if, as the metropolitan population increases, rich households start residing in areas previously inhabited by poor households.

4. SIMULATION FRAMEWORK

The analysis of this paper is to simulate the equilibrium structure of a metropolitan area as its population grows. The utility function of a household with endowed income M is specified as:

$$c + AA\, M^{\alpha} \log_e l + BB\, M^{\beta} \log_e h - CC\, M^{\gamma} \frac{1}{g^{\varphi}}$$

where *AA, BB* and *CC* are constants. We chose this functional form - with separability, with a linear dependence on the consumption of the numeraire and with a logarithmic dependence on land and housing capital - for computational tractability.[17] We chose the parameter values of α, β, γ, φ, *AA, BB* and *CC* to match elasticities and income shares as:[18]

Table 1: Assumed elasticities and expenditure shares

	income elasticity	price elasticity	expenditure shares
Land	.4 [a]	-1 [b]	.03 [c]
Housing capital	1.2 [d]	-1[e]	.15 [f]
Public service	0.7 [g]	-0.5 [h]	.09 [i]

[a] Muth (1971) estimates the income elasticity of demand for land as: 0.328.
Straszheim (1975) estimates income elasticity of lot size as: 0.345. Cheshire and Sheppard (1998) use U.K. data and estimate income elasticity of land area to be in range: 1.678 - 3.755 . Glaeser et al (2000) estimate income elasticity of land demand as being in range: 0.1 - 0.4. We use the upper-value of the Glaeser et al. estimate.

[b] The logarithmic dependence of utility on land implies a price elasticity of: -1. This is within the range determined by studies, viz. Muth (1971) estimates the price elasticity of demand for land as: -0.512. Straszheim (1975) estimates price elasticity of lot size as: - 1.072.
King (1976) estimates price elasticity for "site characteristics" (which include land) as: -0.82. Cheshire and Sheppard (1998) use U.K. data and estimate price elasticity of land area to be between -0.804 and -1.533 Gyourko and Voith (2001) use suburban Philadelphian data to estimate the price elasticity as: -1.64.

[c] From National Income and Product Accounts 2000, Table 2.1: the Compensation of Employees is 5783 ($b). From Table 2.5.5: housing expenditures (including imputed rent) is 1006 ($b). We consider this to be "total housing" comprised of housing capital plus land. Therefore expenditure on housing capital plus land as share of "income" is .17.

To determine land value as a share of house value: Muth (1971) estimates that land expenditure as fraction of house price is: 0.18. Gyourko and Voith (2001) find that for the Philadelphia suburbs land as a share of house value is: 0.15. We accept this value.

Therefore land expenditure as a share of income is: 0.15 x 0.17 = 0.03.

Similarly, housing capital as a share of income is: 0.85 x 0.17 = 0.15.

[d] Muth (1971) estimates the income elasticity of housing capital as: 0.778. McMillan (1979) estimates the income elasticity of internal space as: 1.20. Cheshire and Sheppard (1998) use U.K. data and estimate income elasticity of internal space to be in range: 1.592 - 1.751. We use McMillan's estimate as it lies between the other two.

[e] The logarithmic dependence of utility on housing capital implies a price elasticity of: -1. This is within the range determined by studies, viz. Muth (1971) estimates the price elasticity of demand for housing capital as: - 1.0388. Straszheim (1975) estimates price elasticity of rooms as: -0.078. King (1976) estimates price elasticity for interior space as -.14 . McMillan (1979) estimates the price elasticity for interior space as: -3.47. Cheshire and Sheppard (1998) use U.K. data and estimate price elasticity of internal space to be in range between -0.515 and -1.216.

[f] See note *c*.

[g] Inman (1979) reviews the literature and finds most estimates for the income elasticity of public spending lie in range: 0.2 - 0.7. More recent community-based studies find income elasticities which exceed 1 e..g. Schwab and Zampelli (1987) find the income elasticity for public safety to be: 1.0-1.2 . Dynarski et al. (1989) find the income elasticity for education to be: 3.65. Duncombe (1991) finds the income elasticity for fire protection to be: 1.3. In view of the higher estimates of recent studies, we take the upper value of Inman's range.

[h] Inman (1979) reviews the literature and finds most estimates for the price elasticity of public spending lie in the range between -0.1 and -0.5. We use the upper (absolute) value of Inman's range.

[i] From National Income and Product Accounts 2000, Table 2.1: the Compensation of Employees is 5783 ($b). From Table 3.21: Current Tax Receipts of Local Government is 353 ($b). Therefore local tax receipts as share of "income" is: 0.06. But local government also receives financing from state and federal government, so that its expenditure (from Table 3.21) is 715 ($b), and local expenditure as share of "income" is: 0.12. We use the mid value: 0.09.

These values imply parameter values as:

$\alpha = .4; \ \beta = 1.2; \ \gamma = 1.4; , \varphi = 1; AA = 20, BB = .017$ and $CC = .003$.

We use stylized data (suitably rounded) to set the other parameter values of the model as:

Table 2: simulation parameters

Parameter description	Parameter name	Parameter value
Fraction of households who are poor		.5
City's jurisdictional boundary (miles)[1]	B	8
Income of poor households ($ per year) [2]	M_1	25 000
Income of rich households ($ per year) [2]	M_2	75 000
Commute time per mile as fraction of work day [3]	k	0.0125
Reservation price of land ($ per acre per year) [4]	r_0	2 000
Price of housing capital ($ per unit) [5]	p	2 000
Price of public service ($ per unit) [5]	p	2 000
Fraction of government expenditure financed by the property tax [6]	λ	.4
Fraction of land area used for housing [7]	θ	.4

[1] From Census of Population and Housing 2000, Population and Housing Unit Counts, Table 34: the average inner city area of all CMSA/MSAs with over 1 million residents is 196 (sq. miles). Considering each inner city to be a circle, $\pi B^2 = 196$ or $B=7.9$ (miles).

[2] From Money Income in the United States 2000, Table A.1: the 25th percentile household income is inferred to be 22 000 ($) and the 75th percentile household income is inferred to be 73 000 ($).

[3] Based on round-trip speed of 20 mph and 8 hour working day.

[4] Glaeser, Gyourko and Saks (2005) report a range of land prices in 21 metropolitan areas of $\$_{2000}$ 0.13-4.1 ($ per sq. ft) or $\$_{2000}$ 5663-178598 ($ per acre); using an annual interest rate of .04, this translates into the rental cost of land being between 226 and 7144 ($ per acre). This range is for the land price over a whole metropolitan area and we therefore use a value in the lower part of the range as the value for undeveloped land at the metropolitan boundary.

Gyourko and Voith (2001) show that between 1972-1997 land prices in a developed Philadelphia suburb averaged around 0.9 $\$_{1990}$/sq ft or 51652 $\$_{2000}$ per acre. Using an interest rate of 0.04, this translates into the rental cost of land being 2066 $\$_{2000}$ per acre.

[5] This value is normalized to equal the reservation price of land.

[6] From Census of Governments 1992 Volume 2 Number 1, Table 4: the gross assessed value of residential properties were 2 511 599 ($m) and the gross assessed value of commercial/industrial properties were 997 462 ($m). 1986 was the last census which reported these values. If this division of taxable property was maintained in 2000 and if the tax rate on commercial/industrial property tax rate is 1.757 times as high as the property tax rate on residential property (Table 5, 50-State Property Tax Comparison Study, National Taxpayers Conference, 2006), the fraction of the property tax paid by residences is 2511599 /(2511599 + 997462(1.757)) = 0.589.

From 2000 National Income and Product Accounts, Table 3.21: Property Tax Revenue (246.8 ($b)) as fraction of Local Government Current Tax Receipts (352.6 ($b)) = 0.7.

Hence residential property tax receipts as fraction of locally-raised revenue = 0.589 x 0.7 = .41.

[7] From Census of Population and Housing 2000, Table 34: the density of housing units in central cities is 1126.4 (housing units per sq. mile). From American Housing Survey for the United States 2005, Table 1B-3: the median lot size of 1-unit structures in central cities is 0.21 (acres). Treating the median as the average, the area of housing units per square mile is 1126.4 x 0.21 / 640 = .37

Note that, in a departure from our earlier work, we explicitly take account of the fact that only 40% of a city's developed land area is devoted to housing with the remaining land being used for businesses and infrastructure.

5. EQUILIBRIA OF THE METROPOLITAN AREA

In this section we describe the static equilibria as the metropolitan population increases but the city's jurisdictional boundary B remains fixed. We focus on the case $\lambda = .4$ because it represents current "average" practice in the U.S; in Section 5 we will consider the cases of $\lambda = 0$ and $\lambda = 1$. There are many potential equilibrium configurations corresponding to which income class forms the majority in the city and whether the city includes one or both income classes, which income class forms the majority in the suburb and whether the suburb includes one or both income classes, and whether there is undeveloped land in the city.[19] Instead of discussing all the possible equilibrium configurations, we present below only the equilibria actually found in our simulations.

5.1 Very small metropolitan populations: the single city

The boundaries and populations of equilibria with the "single city" configuration are shown in Table 3. Rents, public service levels, tax rates and average lot sizes are shown in Table 6 in Appendix A.

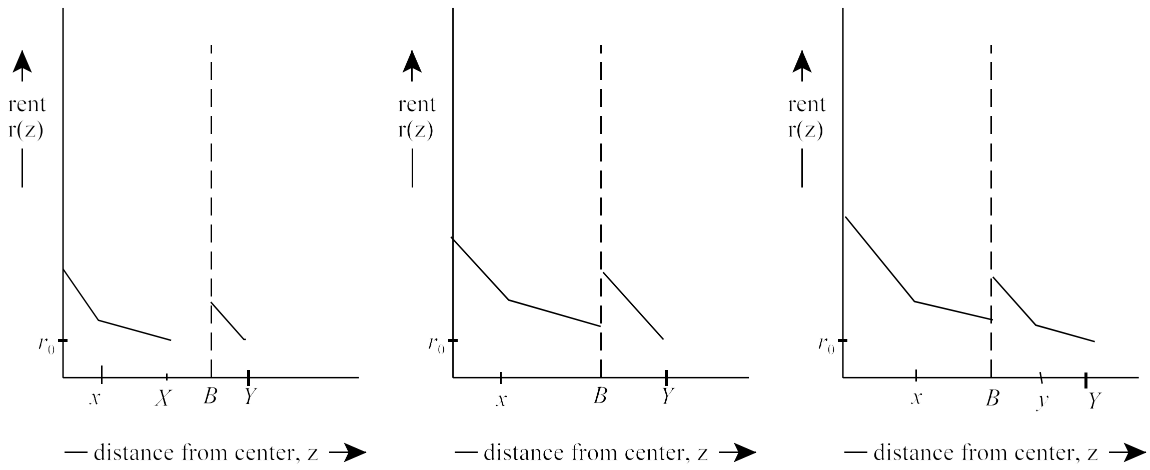
Metropolitan Population *N*	Equilibrium Configuration	Boundary between rich and poor in city, *x* (miles)	Limit of city development *X* (miles)	Boundary between rich and poor in suburb *y* (miles)	Limit of suburb development *Y* (miles)	Number of rich households in city	Number of poor households in city	Number of rich households in suburb	Number of poor households in suburb
7000	single city	1.47	2.04			3 500	3 500		
10 000	single city	1.69	2.38			5 000	5 000		
20 000	single city	2.18	3.18			10 000	10 000		
40 000	single city	2.73	4.19			20 000	20 000		
80 000	single city	3.30	5.42			40 000	40 000		

Table 3: equilibria with the "single city" configuration and $\lambda = .4$

Table 3 shows that, as the metropolitan population increases, *x* and *X* move outwards; from Table 6 rents increase and average plot sizes fall.[20]

As rents rise in the city, the suburb becomes increasingly attractive to rich households. At a metropolitan population slightly larger than *N = 80 000* (actually at *N = 80 016*), rich households can achieve the same utility in the suburb as in the city: as discussed in Section 3, by moving to the suburb, a rich household can vote a higher public service level, pay a lower rent and avoid paying the transfer to poor households which is implicit in the property tax. If the population increases further, the suburb becomes inhabited with some rich households and the "single city" ceases to be an equilibrium configuration. This occurs while there is still undeveloped land ($X < B$). Although we will look at the different equilibrium configurations, this establishes our main result: while there is still undeveloped city land, rich households "jump" to the suburb, establishing themselves as the majority there. This leaves poor households as the majority in the city - an equilibrium configuration which is maintained as the metropolitan area continues to grow.

5.2 Equilibria with both jurisdictions occupied: the city having a majority of poor households



Configuration A Configuration B Configuration C

Figure 5: Equilibrium configurations with city having majority of poor households

Above a critical population both jurisdictions are inhabited. We first consider the equilibrium configurations in which the city's majority is poor. We describe these equilibria in considerable detail because, as noted in the Introduction, poor households tend to concentrate in the cities of most U.S. metropolitan areas. The equilibria are illustrated in Figure 5 and their boundaries and populations are shown in Table 4 below. Rents, public services, tax rates and lot sizes are shown in Table 7 in Appendix A.

In Configuration A all poor households live in the city where they form the majority and vote a low public service level. Some rich households live in the suburb where they vote a high public service level and some rich households live in the city where they benefit from the low commuting; the rent differential balances these two forces. As the metropolitan population increases, "new" rich households divide themselves between the city and the suburb, and *x* and *X*

Metropolitan Population *N*	Equilibrium Configuration	Boundary between rich and poor in city, *x* (miles)	Limit of city development *X* (miles)	Boundary between rich and poor in suburb *y* (miles)	Limit of suburb development *Y* (miles)	Number of rich households in city	Number of poor households in city	Number of rich households in suburb	Number of poor households in suburb
7000									
10 000									
20 000									
40 000									
50 000									
80 000									
90 000	A	3.31	5.59		8.15	42 187	45 000	2 813	
100 000	A	3.32	5.75		8.29	44 439	50 000	5 651	
150 000	A	3.36	6.44		8.85	54 982	75 000	20 018	
200 000	A	3.39	6.99		9.28	65 484	100 000	34 516	
250 000	A	3.42	7.46		9.62	75 949	125 000	49 051	
300 000	A	3.45	7.86		9.90	86 410	150 000	63 590	
350 000	B	3.41	8.00		10.17	95 371	175 000	79 629	
400 000	B	3.34	8.00		10.42	103 325	200 000	96 675	
500 000	B	3.24	8.00		10.84	118 687	250 000	131 313	
600 000	B	3.16	8.00		11.18	133 504	300 000	166 496	
700 000	B	3.10	8.00		11.47	147 917	350 000	202 083	
800 000	B	3.04	8.00		11.73	162 019	400 000	237 918	
900 000	B	3.00	8.00		11.95	175 871	450 000	274 129	
1 000 000	B	2.97	8.00		12.15	189 519	500 000	310 481	
2 000 000	B	2.77	8.00		13.49	319 626	1 000 000	680 374	
3 000 000	B	2.68	8.00		14.27	444 237	1 500 000	1 055 763	
4 000 000	C	2.77	8.00	14.51	14.96	631 850	1 976 039	1 368 150	23 961
5 000 000	C	2.97	8.00	14.28	15.71	919 317	2 409 883	1 580 683	90 117
10 000 000	C	3.34	8.00	13.77	18.10	2 351 220	4 551 279	2 648 780	448 721
15 000 000	C	3.47	8.00	13.56	19.53	3 811 910	6 669 460	3 688 090	830 540
20 000 000	C	3.54	8.00	13.43	20.55	5 290 770	8 776 950	4 709 230	1 223 050
25 000 000	C	3.58	8.00	13.35	21.34	6 780 860	10 878 300	5 719 140	1 621 700
30 000 000	C	3.61	8.00	13.29	22.00	8 278 490	12 975 700	6 721 510	2 024 330

Table 4: equilibria with the city having a majority of poor households and with $\lambda = .4$

move outward. At a critical population slightly above 300 000, all city land is developed and the equilibrium configuration shifts to Configuration B.

With Configuration B: as the population continues to increase, some of the additional rich households locate in the city and some locate in the suburb. In Configuration A, the increasing population of the city was accommodated by the limit of development X moving outwards. Now, however, the city is fully developed and hence the increasing number of poor households pushes the class-boundary x away from the city's jurisdictional boundary. Parts of the city which had been inhabited by rich households are now inhabited by poor households. The larger rich population in the city is able to be accommodated in a smaller area because the higher rents induce smaller land plot sizes.

As the metropolitan population further increases, rents continue to rise in the city and at a critical population size, which is slightly above 3 000 000 , poor households are able to achieve the same utility at the edge of the suburb as they achieve in the city. This is Configuration C with both income classes resident in both jurisdictions.

Configuration C is the equilibrium which is discussed extensively in de Bartolome and Ross (2003, 2004, 2007) where it is termed "income mixing". As the population continues to increase, some "new" poor households locate in the suburb, allowing the class boundary x to move outwards. Parts of the city which had been inhabited by poor households revert to rich households. This is our explanation for the phenomena which is popularly termed "gentrification": rents in the city are sufficiently high that some poor households locate in the suburb, allowing rich households to live in areas of the city which had previously been inhabited by poor households.

As the metropolitan population increases and the equilibrium configuration changes from Configuration A through Configuration B, the proportion of the city's population which is poor increases. Because the property tax is being partially used to finance the city's public service, the increasing proportion of poor households increases the tax-price of the public service. As shown in Table 7, this causes the public service voted in the city to deteriorate and the city's property tax rate to increase. However, the situation reverses when the configuration changes to Configuration C. With poor households now locating in the suburb, the proportion of the city's population which is poor decreases, lowering the tax-price of the public service and causing the city's public service to "rebound" and the city's property tax rate to decrease.

Rents paid by rich households exceed those paid by poor households but - beyond a metropolitan population of 400 000 - most rich households are located in the suburb where rents are relatively low. Hence the average lot size of rich households exceeds that of poor households until the metropolitan population exceeds 10 000 000.

5.3 Equilibria with both jurisdictions occupied: the city having majority of rich households

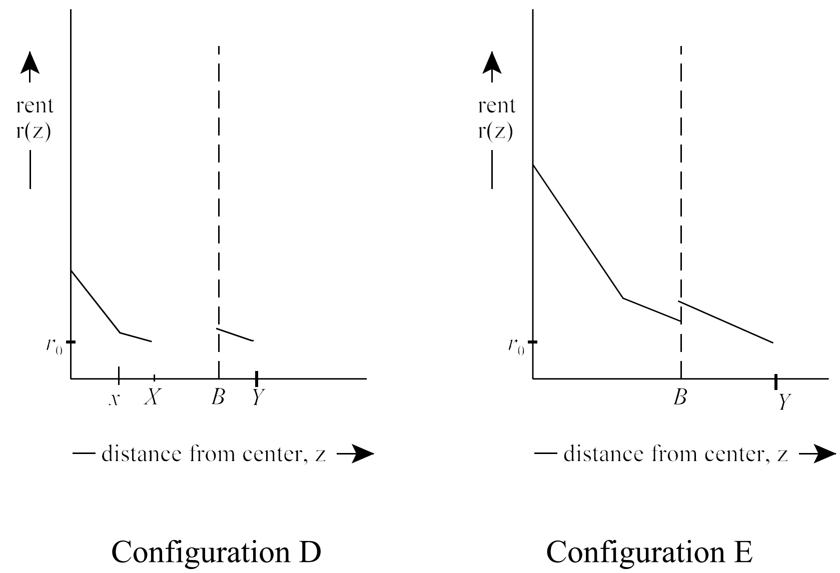


Figure 6: Equilibrium configurations with city having majority of rich households and λ=.4.

Section 5.2 has discussed equilibria with poor households being the majority in the city. With both jurisdictions being occupied, there are alternative equilibria in which rich households form the majority in the city, voting a high public service there. The equilibria are illustrated in Figure 6 and their boundaries and populations are shown in Table 5 below. Rents, public services, tax rates and lot sizes are shown in Table 8 in Appendix A.

Configuration D is the analogue to Configuration A. The city is inhabited by both income classes but now rich households are the majority, and the suburb is inhabited only by poor households. For poor households living in the suburb, the cost of the longer commute is balanced by the benefit of being able to vote a lower public service. There is undeveloped city land.

As the metropolitan population grows, "new" poor households distribute themselves between the city and the suburb, and the boundaries *x* and *X* move outwards until the limit of development reaches the city's jurisdictional boundary and Configuration E is reached.

Configuration E is the analogue of Configuration B. As the metropolitan population continues to grow: some “new” poor households locate in the suburb but all “new” rich households locate in the city, pushing the class boundary x outwards. The increasing population in the city is accommodated by the smaller lot sizes induced by the increase in rents. Rich households make up an increasing proportion of the city’s population, increasing the implicit subsidy to each poor household; this keeps poor households in the city and this configuration is maintained at all reasonable metropolitan populations.[21]

Metropolitan Population N	Equilibrium Configuration	Boundary between rich and poor in city, x (miles)	Limit of city development X (miles)	Boundary between rich and poor in suburb y (miles)	Limit of suburb development Y (miles)	Number of rich households in city	Number of poor households in city	Number of rich households in suburb	Number of poor households in suburb
7000									
10 000									
20 000									
40 000									
50 000									
80 000									
90 000									
100 000									
150 000									
200 000									
250 000	D	4.12	7.60		8.12	125 000	121 350		3 650
300 000	D	4.32	7.96		8.44	150 000	135 350		14 650
350 000	E	4.43	8.00		8.80	175 000	146 146		28 854
400 000	E	4.52	8.00		9.14	200 000	156 221		43 779
500 000	E	4.66	8.00		9.73	250 000	175 736		74 264
600 000	E	4.77	8.00		10.22	300 000	194 634		105 366
700 000	E	4.85	8.00		10.66	350 000	213 086		136 914
800 000	E	4.92	8.00		11.04	400 000	231 199		168 801
900 000	E	4.97	8.00		11.39	450 000	249 047		200 953
1 000 000	E	5.02	8.00		11.70	500 000	266 679		233 321
2 000 000	E	5.27	8.00		13.87	1 000 000	436 405		563 595
3 000 000	E	5.38	8.00		15.20	1 500 000	600 496		899 504
4 000 000	E	5.44	8.00		16.17	2 000 000	762 109		1 237 891
5 000 000	E	5.48	8.00		16.93	2 500 000	922 314		1 577 686
10 000 000	E	5.57	8.00		19.36	5 000 000	1 713 730		3 286 270
15 000 000	E	5.61	8.00		20.80	7 500 000	2 498 370		5 001 630
20 000 000	E	5.62	8.00		21.83	10 000 000	3 280 150		6 719 850
25 000 000	E	5.63	8.00		22.64	12 500 000	4 060 340		8 439 660
30 000 000	E	5.64	8.00		23.29	15 000 000	4 839 510		10 160 490

Table 5: equilibria with the city having a majority of rich households and with $\lambda = .4$

5.4 Further discussion of the growth path

Tables 3 and 4 show that the "single city" continues to be the equilibrium configuration until slightly above *80 000* or until $N = 80\ 016$. As the metropolitan population increases beyond $N = 80\ 016$ some rich households locate in the suburb: the equilibrium changes continuously from the configuration of the "single city" to Configuration A (x, X and Y change continuously through the transition). Viewing the metropolitan area's path as a continuous sequence of static equilibria, rich households "jump" over undeveloped land to the suburb before poor households "spill" into the suburb. Rich households win the race to the suburb leaving poor households in control of the city.

Figure 6 shows this in a different way. We continue to focus on the case with $\lambda = .4$. The central axis shows the metropolitan population N. The equilibrium configuration (at each N) in which poor households are the city's majority is shown above the central axis, and the equilibrium configuration in which rich households are the city's majority is shown below the central axis. For example: for metropolitan populations between 80 016 and 317 579, there is an equilibrium with poor households being a majority in the city and it has Configuration A.

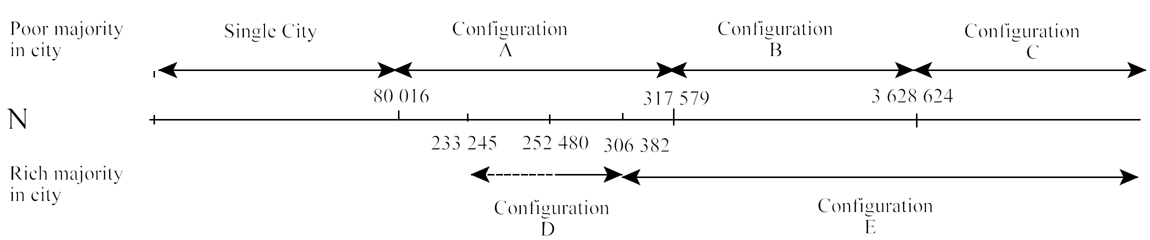


Figure 6: summary of equilibrium configurations as N changes with $\lambda = .4$

Figure 6 shows that the “single city” is the only equilibrium configuration at small metropolitan populations but it ceases to be an equilibrium configuration when the metropolitan population reaches *80 016.* Between populations of *80 016* and *233 245*, Configuration A is the only equilibrium and hence it is selected when the “single city” equilibrium breaks down. Our assumption about continuity implies that, once this configuration - with the poor being the majority in the city - has been established, it is maintained

A configuration with rich households being the majority in the city (Configuration D) does not exist until the metropolitan population is 233 245. If rich households were forbidden to locate in the suburb, the “single city” would become fully developed and poor households would “spill” into the suburb when the metropolitan population reached 252 480. As indicated in Section 3, as poor households spilled into the suburb, rich households would become the city’s majority. They would vote a high public service in the city, causing additional poor households to locate to the suburb, creating undeveloped land in the city. This would correspond to an equilibrium with the form of Configuration D. Hence, although Configuration D exists as a static equilibrium configuration at populations above 233 245, even if the poor do not to “jump” to the suburb, it cannot be reached from the “single city” until the metropolitan population reaches 252 480. This is shown in Figure 6 by the arrow covering the populations for which Configuration D is an equilibrium being dashed for populations between 233 245 and 252 80.

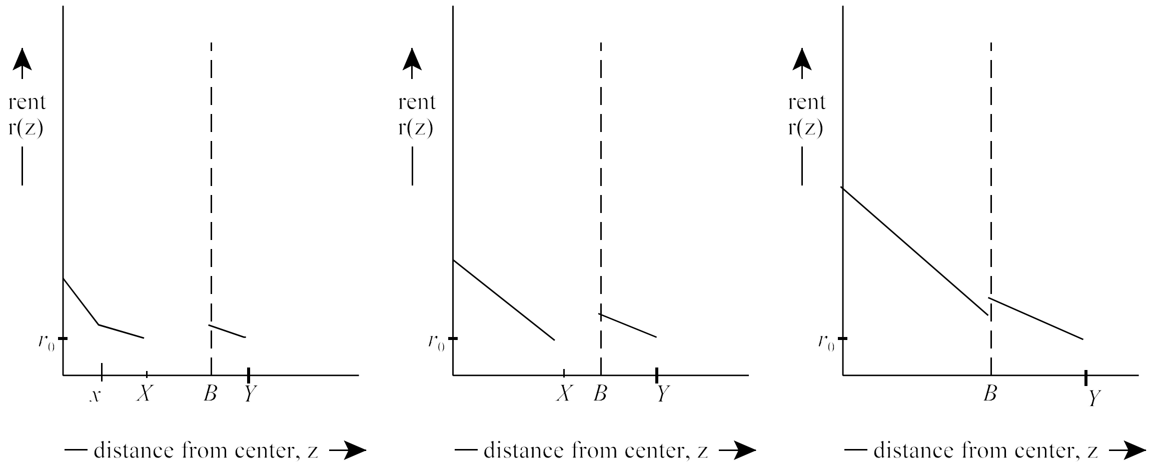
5.5 What is causing the rich to "jump" to the suburb? The pure residence tax.

In our model, rich households "jump" to the suburb before poor households "spill" into the suburb. The net effect of the commuting cost and inelastic land demand is to push rich households towards the city. If jurisdictions financed their expenditures using only a residence tax, the only force pushing rich households towards the suburb would be the fiscal force - by moving to the suburb, a rich household can vote a higher pubic service than is provided by the city. However, the partial use of the property tax introduces additional forces. Implicit in the property tax is a transfer from rich to poor households. Because a rich household "jumping" to the suburb avoids this transfer, this aspect of the property tax reinforces the force pushing the rich household to jump. However the property tax has another aspect which "holds the rich household back." Because it lowers the tax price of the public service to poor households (relative to the pure residence tax), it raises the public service voted in the city which makes the city more attractive to rich households and delays their "jump" to the suburb.

To determine if it is the fiscal effect *per se* which is causing rich households to "jump" (and hence creating cities in which poor households congregate), we reran the simulations using a pure residence tax or setting $\lambda = 0$. The characteristics of the computed equilibria are shown in Appendix B; the characteristics of the "single city" are shown in Table 9.With $\lambda = 0$ the "single city" ceases to be an equilibrium when $N = 44\ 103$ and while there is undeveloped land in the city; at this population a rich household can achieve the same utility whether he locates in the city or in the suburb. With the partial use of the property tax, the "single city" ceased to be an equilibrium when $N = 80\ 016$. Therefore the primary force causing rich households to "jump" to the suburb is the difference in the public service levels, and the property tax, by inducing a higher public service level in the city, delays the “jump”.

With $\lambda = 0$ and with both jurisdictions being occupied, the equilibrium configurations in which poor households are the majority in the city are the same configurations as were illustrated in Figure 5; the characteristics of the equilibria at different metropolitan populations are shown in Table 10 in Appendix B. When the metropolitan population exceeds *44 103*, the equilibrium configuration is Configuration A, which is reached from the “single city” by a continuous change in the boundaries x, X and Y.

The property tax raises the after-tax rent and reduces lot sizes. Hence removing the property tax or setting $\lambda = 0$ causes the city to fill up faster and the shift from Configuration A to Configuration B occurs at a lower metropolitan population. In considering the shift from Configuration B to Configuration C: as noted earlier, the property tax includes an implicit transfer from rich households to poor households in jurisdictions in which both are present. This makes it beneficial for poor households to locate in the suburb when it is controlled by rich households. In consequence, removing the property tax or setting $\lambda = 0$ makes the suburb less attractive to poor households and the shift from Configuration B to Configuration C occurs at a higher metropolitan population. With $\lambda = 0$, gentrification (interpreted as x increasing as the metropolitan population increases) occurs later.



Configuration D   Configuration F   Configuration G

Figure 7: Equilibrium configurations with the city having a majority of rich households and λ=0.

Figure 7 shows the equilibria in which rich households are the majority in the city (and with $\lambda = 0$); characteristics of these equilibria are shown in Table 11 in Appendix B. The pure residence tax avoids the transfer from rich to poor households implicit with the property tax, and therefore reduces the incentive for poor households to locate in jurisdictions controlled by rich households. Therefore Configuration D occurs earlier if the partial property tax is replaced with a pure residence tax.

With $\lambda = 0$ and with Configuration D: as the metropolitan population increases, the increasing number of rich households in the city pushes the class boundary *x* outwards. The public services in each jurisdiction are unchanged. Considering poor households at *X* and *Y*: they pay the same rent. Therefore, as the population increases, the commuting advantage of a poor household at *X* relative to that of a poor household at *Y* must be maintained and this is achieved by the limits of development in the city (*X*) and in the suburb (*Y*) moving outwards at the same rate. The bigger area traced out by the change in radius in the suburb means that some poor households who previously lived in the city now locate in the suburb so that the number of poor households in the city actually declines. At a critical population just less than 20 000 all poor households live in the suburb and complete "income sorting" is achieved while there is still undeveloped city land. This configuration is denoted Configuration F.

With Configuration F (and $\lambda = 0$): as the metropolitan population increases, the "new" rich households locate in the city and the limit of development in the city continues to move outwards. At a metropolitan population just above 800 000, all land in the city is developed. This configuration is denoted Configuration G.

Figure 8 repeats the analysis of Figure 7 but with $\lambda = 0$.

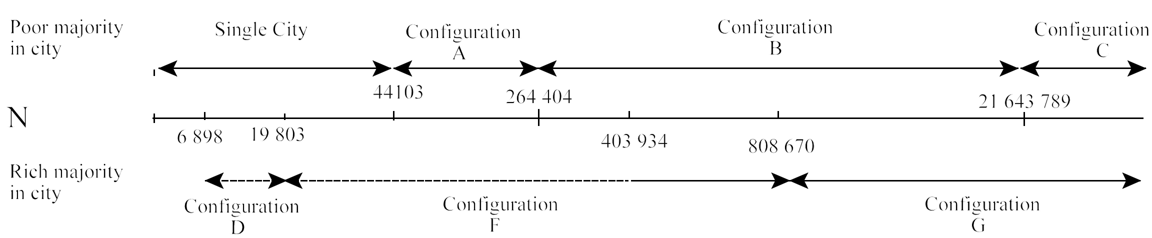


Figure 7: summary of equilibrium configurations as N changes with $\lambda = 0$

Figure 8 shows that, at metropolitan populations for which the "single city" is an equilibrium configuration, there are also equilibria with Configurations D and F (the rich being the majority in the city). However, these latter configurations require that some poor households locate in the suburb - a location choice which, with undeveloped city land, would be unfavorable to them. Put differently, the incentives faced by poor households prevent Configurations D and E being reached from the "single city" configuration with undeveloped land. The "single city" ceases to be an equilibrium before poor households spill into the suburb (if rich households were forbidden to jump to the suburb, this would occur at $N = 403\ 934$).

5.6. $\lambda = 1$ *and robustness*

Similar results are obtained with $\lambda = 1$ although the details differ. In particular the full use of the property tax causes the public service voted by poor households in the "single city" to rise and delays the "jump" of rich households to the suburb: this now occurs when $N = 132\ 730$. When the "jump" occurs there is still undeveloped city land.[22] With the public service being fully financed by the property tax, the incentive for a poor household to live in a jurisdiction containing rich households is increased: as rich households "jump", poor households "follow": x, X, and Y change discontinuously. Hence the configurations of the subsequent equilibria differ from those illustrated in Figure 5.

In our framework, if x, X and Y change discontinuously, any configuration which is an equilibrium configuration can potentially be selected. However, with $\lambda = 1$, at $N = 132\ 730$, the only equilibrium with both jurisdictions occupied has only poor households living in the city, and rich and poor households living in the suburb. A configuration with rich households living in the city does not become a equilibrium until $N = 4\ 332\ 279$.[23] Therefore, with $\lambda = 1$, when the "single city" ceases to be an equilibrium, the configuration in which poor households are the majority in the city is selected; all rich households locate in the suburb and there is undeveloped city land. As the population increases, further changes are continuous. At a population of $N = 3\ 503\ 550$ the city becomes fully developed: the configuration continues to have only poor households living in the city, and rich and poor households living in the suburb.

The above discussion has considered variation in the tax parameter λ. Our main result - that rich households "jump into" the suburb before poor households "spill into" the suburb - is robust if each of the other parameter values is changed by ± 20%. However, the analysis on gentrification is less robust because, when some parameter values change by ± 20%, Configuration C ceases to be an equilibrium configuration.

6. CONCLUSION

In a monocentric urban model with two jurisdictions - an inner city and a surrounding suburb - there tend to be two equilibria: one in which poor households are the inner city's majority and one in which rich households are the inner city's majority. Our simulation suggests that the growth path of the metropolitan area selects the equilibrium in which poor households are the inner city's majority because rich households migrate to form a new jurisdiction in the suburb while there is still undeveloped land in the city. At large metropolitan populations, the equilibrium has both income classes living in both jurisdictions; further population growth causes gentrification as some rich households locate in areas in the city which were previously inhabited by poor households.

Our model has focused on the U.S. experience. In doing so, we have assumed that separate jurisdictions have considerable autonomy, leading to considerable variation in the public service level across jurisdictions. The equilibrium in which rich households congregate in the suburb is selected because of their high willingness to pay for their preferred public service. Breuckner, Thisse and Zenou (1999) note that many European cities have higher average income in the inner city than in the suburbs. We believe that an important difference between the U.S. and Europe is that in Europe there is less variation in the public service level across jurisdictions and less reliance on the property tax.[24] In our model, if a regional government is introduced which prevents large differences in the public service being established between jurisdictions, rich households have a smaller incentive to "jump" over undeveloped city land to form a new jurisdiction in the suburb. If the allowed difference is sufficiently small, the equilibrium growth path may have poor households "spilling" into the suburb and rich households forming the inner city's majority. Thus it seems that, by adding a regional government to our model and imbuing it with different powers or roles, we might be able to explain the difference between the U.S. and the European experiences. This is an issue we intend to explore in future research.

APPENDIX A: ADDITIONAL JURISDICTIONAL CHARACTERISTICS WITH λ = .4

Metropolitan Population N	Equilibrium Configuration	rent at city's center r(0) ($/acre)	rent at city side of jurisdiction boundary $r(B^-)$ ($/acre)	rent at suburb side of jurisdiction coundary $r(B^+)$ ($/acre)	City public service g_c	City property tax rate t_c	Suburb public service g_s	Suburb property tax rate t_s	Average lot size of poor households (acres)	Average lot size of rich households (acres)
7 000	single city	5 054			1.65	0.17			0.46	0.50
10 000	single city	5 864			1.65	0.17			0.45	0.46
20 000	single city	8 253			1.65	0.17			0.43	0.38
40 000	single city	12 482			1.65	0.17			0.41	0.30
80 000	single city	20 225			1.65	0.17			0.37	0.22

Table 6: rents, public service levels, tax rates and lot sizes of "single city" with λ = .4

Metropolitan Population N	Equilibrium Configuration	Rent at city's center r(0) ($/acre)	Rent at city side of jurisdiction boundary $r(B^-)$ ($/acre)	Rent at suburb side of jurisdiction boundary $r(B^+)$ ($/acre)	City public service g_c	City property tax rate t_c	Suburb public service g_s	Suburb property tax rate t_s	Average lot size of poor households (acres)	Average lot size of rich households (acres)
7 000										
10 000										
20 000										
40 000										
50 000										
80 000										
90 000	A	21 226		2 166	1.65	0.17	3.17	0.22	0.36	0.24
100 000	A	22 213		2 330	1.64	0.18	3.17	0.22	0.36	0.25
150 000	A	27 013		3 135	1.63	0.18	3.17	0.22	0.32	0.28
200 000	A	31 682		3 918	1.63	0.19	3.17	0.22	0.30	0.27
250 000	A	36 279		4 685	1.62	0.20	3.17	0.22	0.28	0.26
300 000	A	40 832		5 441	1.62	0.20	3.17	0.22	0.27	0.25
350 000	B	45 487	2 172	6 267	1.61	0.20	3.17	0.22	0.24	0.23
400 000	B	50 144	2 436	7 136	1.61	0.20	3.17	0.22	0.21	0.22
500 000	B	59 268	2 956	8 885	1.61	0.21	3.17	0.22	0.17	0.21
600 000	B	68 202	3 471	10 646	1.60	0.21	3.17	0.22	0.14	0.19
700 000	B	76 995	3 981	12 416	1.60	0.22	3.17	0.22	0.13	0.18
800 000	B	85 678	4 488	14 193	1.60	0.22	3.17	0.22	0.11	0.17
900 000	B	94 275	4 992	15 977	1.59	0.22	3.17	0.22	0.10	0.16
1 000 000	B	102 800	5 494	17 765	1.59	0.22	3.17	0.22	0.09	0.15
2 000 000	B	185 812	10 441	35 817	1.58	0.23	3.17	0.22	0.06	0.10
3 000 000	B	266 893	15 328	54 021	1.58	0.24	3.17	0.22	0.03	0.08
4 000 000	C	367 259	20 635	69 375	1.58	0.23	3.16	0.23	0.03	0.06
5 000 000	C	497 418	26 532	80 292	1.59	0.22	3.14	0.23	0.03	0.05
10 000 000	C	1 141 070	55 425	135 157	1.61	0.20	3.10	0.25	0.03	0.02
15 000 000	C	1 790 950	84 231	188 678	1.62	0.20	3.08	0.26	0.03	0.01
20 000 000	C	2 445 470	113 057	241 352	1.62	0.20	3.07	0.26	0.02	0.01
25 000 000	C	3 102 930	141 905	293 503	1.62	0.19	3.06	0.26	0.02	0.01
30 000 000	C	3 762 410	170 769	345 302	1.62	0.19	3.05	0.27	0.02	0.01

Table 7: equilibria with the city having a majority of poor households and with $\lambda = .4$

Metropolitan Population *N*	Equilibrium Configuration	Rent at city's center r(0) ($/acre)	Rent at city side of jurisdiction boundary $r(B^-)$ ($/acre)	Rent at suburb side of jurisdiction boundary $r(B^+)$ ($/acre)	City public service g_c	City property tax rate t_c	Suburb public service g_s	Suburb property tax rate t_s	Average lot size of poor households (acres)	Average lot size of rich households (acres)
7 000										
10 000										
20 000										
40 000										
50 000										
80 000										
90 000										
100 000										
150 000										
200 000										
250 000	D	44 984		2 064	2.89	0.34	1.47	0.37	0.27	0.11
300 000	D	52 160		2 253	2.90	0.33	1.47	0.37	0.28	0.10
350 000	E	59 890	2 205	2 489	2.91	0.32	1.47	0.37	0.27	0.09
400 000	E	67 679	2 437	2 728	2.93	0.32	1.47	0.37	0.25	0.08
500 000	E	83 193	2 891	3 200	2.94	0.31	1.47	0.37	0.23	0.07
600 000	E	98 645	3 338	3 663	2.96	0.31	1.47	0.37	0.22	0.06
700 000	E	114 053	3 779	4121	2.97	0.30	1.47	0.37	0.21	0.05
800 000	E	129 427	4 216	4 575	2.97	0.30	1.47	0.37	0.20	0.05
900 000	E	144 775	4 649	5 025	2.98	0.30	1.47	0.37	0.19	0.04
1 000 000	E	160 101	5 079	5 473	2.99	0.29	1.47	0.37	0.18	0.04
2 000 000	E	312 714	9 298	9 863	3.01	0.28	1.47	0.37	0.13	0.02
3 000 000	E	464 781	13 448	14 179	3.02	0.28	1.47	0.37	0.11	0.02
4 000 000	E	616 613	17 567	18 463	3.03	0.28	1.47	0.37	0.09	0.01
5 000 000	E	768 314	21 670	22 729	3.03	0.27	1.47	0.37	0.08	0.01
10 000 000	E	1 525 940	42 073	43 935	3.04	0.27	1.47	0.37	0.06	0.005
15 000 000	E	2 282 950	62 400	65 056	3.04	0.27	1.47	0.37	0.04	0.003
20 000 000	E	3 039 700	82 696	86 141	3.05	0.27	1.47	0.37	0.04	0.003
25 000 000	E	3 796 320	102 974	107 207	3.05	0.27	1.47	0.37	0.03	0.002
30 000 000	E	4 552 850	123 242	128 260	3.05	0.27	1.47	0.37	0.03	0.002

Table 8: equilibria with the city having a majority of rich households and with $\lambda = .4$ (cont)

APPENDIX B: RESULTS WITH USE OF RESIDENCE TAX ONLY ($\lambda = 0$)

Metropolitan Population N	Equilibrium Configuration	Boundary between rich and poor in city, x (miles)	Limit of city development X (miles)	Boundary between rich and poor in suburb y (miles)	Limit of suburb development Y (miles)	Number of rich households in city (n_{2c})	Number of poor households in city (n_{1c})	Number of rich households in suburb (n_{2s})	Number of poor households in suburb (n_{1s})
7000	single city	1.56	2.18			3 500	3 500		
10 000	single city	1.79	2.54			5 000	5 000		
20 000	single city	2.30	3.39			10 000	10 000		
40 000	single city	2.86	4.45			20 000	20 000		

Metropolitan Population N	Equilibrium Configuration	Rent at city's center r(0) ($/acre)	Rent at city side of jurisdiction boundary r(B$^-$) ($/acre)	Rent at suburb side of jurisdiction boundary r(B$^+$) ($/acre)	City public service g_c	City property tax rate t_c	Suburb public service g_s	Suburb property tax rate t_s	Average lot size of poor households (acres)	Average lot size of rich households (acres)
7 000	single city	5 383			1.47				0.53	0.56
10 000	single city	6 296			1.47				0.52	0.52
20 000	single city	9 006			1.47				0.50	0.42
40 000	single city	13 841			1.47				0.47	0.33

Table 9: equilibria of the "single city" with $\lambda = 0$

Metropolitan Population N	Equilibrium Configuration	Boundary between rich and poor in city, x (miles)	Limit of city development X (miles)	Boundary between rich and poor in suburb y (miles)	Limit of suburb development Y (miles)	Number of rich households in city	Number of poor households in city	Number of rich households in suburb	Number of poor households in suburb
7000									
10 000									
20 000									
40 000									
50 000	A	2.97	4.78		8.10	23 495	25 000	1 505	
80 000	A	3.08	5.51		8.53	30 812	40 000	9 187	
90 000	A	3.12	5.71		8.66	33 244	45 000	11 756	
100 000	A	3.15	5.90		8.77	35 674	50 000	14 326	
150 000	A	3.28	6.71		9.25	47 792	75 000	27 208	
200 000	A	3.37	7.34		9.62	59 878	100 000	40 122	
250 000	A	3.45	7.86		9.93	71 944	125 000	53 056	
300 000	B	3.43	8.00		10.21	82 942	150 000	67 058	
350 000	B	3.38	8.00		10.46	93 454	175 000	81546	
400 000	B	3.35	8.00		10.68	103 912	200 000	96 088	
500 000	B	3.30	8.00		11.06	124 710	250 000	125 290	
600 000	B	3.26	8.00		11.37	145 394	300 000	154 606	
700 000	B	3.24	8.00		11.64	165 996	350 000	184 004	
800 000	B	3.21	8.00		11.87	186 538	400 000	213 462	
900 000	B	3.20	8.00		12.08	207 033	450 000	242 967	
1 000 000	B	3.18	8.00		12.27	227 489	500 000	272 511	
2 000 000	B	3.12	8.00		13.53	430 927	1 000 000	569 073	
3 000 000	B	3.09	8.00		14.28	633 433	1 500 000	866 567	
4 000 000	B	3.08	8.00		14.81	835 547	2 000 000	1 164 453	
5 000 000	B	3.07	8.00		15.23	1 037 440	2 500 000	1 462 560	
10 000 000	B	3.06	8.00		16.53	2 045 480	5 000 000	2 954 520	
15 000 000	B	3.05	8.00		17.30	3 052 530	7 500 000	4 447 470	
20 000 000	B	3.04	8.00		17.84	4 059 170	10 000 000	5 940 830	
25 000 000	C	3.05	8.00	17.99	18.51	5 074 680	12 471 404	7 425 320	28 596
30 000 000	C	3.06	8.00	18.00	19.18	6 094 770	14 927 621	8 905 230	72 379

Table 10(a): equilibria with the city having a majority of poor households and with $\lambda = 0$

Metropolitan Population N	Equilibrium Configuration	Rent at city's center r(0) ($/acre)	Rent at city side of jurisdiction boundary $r(B^-)$ ($/acre)	Rent at suburb side of jurisdiction boundary $r(B^+)$ ($/acre)	City public service g_c	City property tax rate t_c	Suburb public service g_s	Suburb property tax rate t_s	Average lot size of poor households (acres)	Average lot size of rich households (acres)
7 000										
10 000										
20 000										
40 000										
50 000	A	15 593		2 109	1.47		3.17		0.45	0.36
80 000	A	19 584		2 649	1.47		3.17		0.42	0.37
90 000	A	20 884		2 825	1.47		3.17		0.41	0.37
100 000	A	22 172		2 999	1.47		3.17		0.40	0.37
150 000	A	28 491		3 853	1.47		3.17		0.37	0.35
200 000	A	34 669		4 689	1.47		3.17		0.34	0.32
250 000	A	40 757		5 513	1.47		3.17		0.32	0.30
300 000	B	47 272	2 247	6 394	1.47		3.17		0.28	0.28
350 000	B	53 953	2 593	7 297	1.47		3.17		0.24	0.26
400 000	B	60 612	2 939	8 198	1.47		3.17		0.21	0.25
500 000	B	73 884	3 629	9 993	1.47		3.17		0.17	0.22
600 000	B	87 110	4 317	11 782	1.47		3.17		0.14	0.20
700 000	B	100 305	5 005	13 567	1.47		3.17		0.12	0.18
800 000	B	113 476	5 692	15 348	1.47		3.17		0.11	0.18
900 000	B	126 629	6 378	17 127	1.47		3.17		0.10	0.16
1 000 000	B	139 767	7 065	18 904	1.47		3.17		0.09	0.16
2 000 000	B	270 713	13 915	36 614	1.47		3.17		0.04	0.10
3 000 000	B	401 305	20 756	54 278	1.47		3.17		0.03	0.08
4 000 000	B	531 749	27 593	71 921	1.47		3.17		0.03	0.07
5 000 000	B	662 111	34 429	89 553	1.47		3.17		0.02	0.06
10 000 000	B	1 313 380	68 591	177 639	1.47		3.17		0.01	0.04
15 000 000	B	1 964 280	102 744	265 675	1.47		3.17		0.01	0.03
20 000 000	B	2 615 030	136 894	353 691	1.47		3.17		0.004	0.02
25 000 000	C	3 265 860	170 838	441 718	1.47		3.17		0.005	0.02
30 000 000	C	3 916 680	204 671	529 744	1.47		3.17		0.005	0.01

Table 10(b): equilibria with the city having a majority of poor households and with $\lambda = 0$ (cont)

Metropolitan Population *N*	Configuration	Boundary between rich and poor in city, *x* (miles)	Limit of city development *X* (miles)	Boundary between rich and poor in suburb *y* (miles)	Limit of suburb development *Y* (miles)	Number of rich households in city	Number of poor households in city	Number of rich households in suburb	Number of poor households in suburb
7000	D	1.57	2.17		8.01	3 500	3 416		84
10 000	D	1.86	2.27		8.11	5 000	2 511		2 489
20 000	F		2.53		8.41	10 000			10 000
40 000	F		3.35		8.77	20 000			20 000
50 000	F		3.63		8.93	25 000			25 000
80 000	F		4.25		9.37	40 000			40 000
90 000	F		4.42		9.50	45 000			45 000
100 000	F		4.57		9.62	50 000			50 000
150 000	F		5.18		10.18	75 000			75 000
200 000	F		5.62		10.66	100 000			100 000
250 000	F		5.99		11.07	125 000			125 000
300 000	F		6.29		11.43	150 000			150 000
350 000	F		6.55		11.76	175 000			175 000
400 000	F		6.77		12.05	200 000			200 000
500 000	F		7.16		12.58	250 000			250 000
600 000	F		7.47		13.03	300 000			300 000
700 000	F		7.74		13.42	350 000			350 000
800 000	F		7.98		13.78	400 000			400 000
900 000	G		8.00		14.11	450 000			450 000
1 000 000	G		8.00		14.40	500 000			500 000
2 000 000	G		8.00		16.48	1 000 000			1 000 000
3 000 000	G		8.00		17.78	1 500 000			1 500 000
4 000 000	G		8.00		18.74	2 000 000			2 000 000
5 000 000	G		8.00		19.49	2 500 000			2 500 000
10 000 000	G		8.00		21.89	5 000 000			5 000 000
15 000 000	G		8.00		23.33	7 500 000			7 500 000
20 000 000	G		8.00		24.36	10 000 000			10 000 000
25 000 000	G		8.00		25.16	12 500 000			12 500 000
30 000 000	G		8.00		25.82	15 000 000			15 000 000

Table11(a): equilibria when the city having a majority of rich households and $\lambda = 0$

Metropolitan Population N	Equilibrium Configuration	Rent at city's center r(0) ($/acre)	Rent at city side of jurisdiction boundary $r(B^-)$ ($/acre)	Rent at suburb side of jurisdiction boundary $r(B^+)$ ($/acre)	City public service g_c	City property tax rate t_c	Suburb public service g_s	Suburb property tax rate t_s	Average lot size of poor households (acres)	Average lot size of rich households (acres)
7 000	D	5 371		2 002	3.17		1.47		0.53	0.56
10 000	D	5 961		2 060	3.17		1.47		0.55	0.56
20 000	F	7 783		2 237	3.17		1.47		0.54	0.54
40 000	F	11 656		2 464	3.17		1.47		0.52	0.45
50 000	F	13 479		2 575	3.17		1.47		0.51	0.42
80 000	F	18 736		2 899	3.17		1.47		0.48	0.36
90 000	F	20 443		3 005	3.17		1.47		0.47	0.35
100 000	F	22 134		3 110	3.17		1.47		0.46	0.34
150 000	F	30 434		3 622	3.17		1.47		0.43	0.29
200 000	F	38 571		4 119	3.17		1.47		0.40	0.25
250 000	F	46 613		4 605	3.17		1.47		0.38	0.23
300 000	F	54 593		5 083	3.17		1.47		0.36	0.21
350 000	F	62 527		5 555	3.17		1.47		0.34	0.20
400 000	F	70 428		6 022	3.17		1.47		0.33	0.18
500 000	F	86 158		6 944	3.17		1.47		0.30	0.16
600000	F	101 818		7 853	3.17		1.47		0.28	0.15
700 000	F	117 430		8 753	3.17		1.47		0.27	0.14
800 000	F	133 005		9 647	3.17		1.47		0.25	0.13
900 000	G	149 528	2 226	10 534	3.17		1.47		0.24	0.11
1 000 000	G	166 142	2 473	11 417	3.17		1.47		0.23	0.10
2 000 000	G	332 284	4 946	20 092	3.17		1.47		0.17	0.05
3 000 000	G	498 426	7 420	28 635	3.17		1.47		0.14	0.03
4 000 000	G	664 568	9 893	37 118	3.17		1.47		0.12	0.03
5 000 000	G	830 170	12 366	45 567	3.17		1.47		0.10	0.02
10 000 000	G	1 661 420	24 732	87 578	3.17		1.47		0.07	0.01
15 000 000	G	2 492 130	37 098	129 422	3.17		1.47		0.05	0.007
20 000 000	G	3 322 840	49 464	171 197	3.17		1.47		0.04	0.005
25 000 000	G	4 153 550	61 830	212 934	3.17		1.47		0.04	0.004
30 000 000	G	4 984 260	74 196	254 645	3.17		1.47		.03	.003

Table 11(b): equilibria with the city having a majority of rich households and $\lambda = 0$ (cont.)

REFERENCES

Alonso, W., (1964), *Location and land use: toward a general theory of land rent*. Cambridge: Harvard University Press.

Arnott, R. J. and F. D. Lewis, (1979), "The transition to urban land use," *Journal of Political Economy* 87, 161-170.

Berglas, E., (1976a), "On the theory of clubs," *Papers and Proceedings of the American Economic Association* 66, 116-121.

Berglas, E., (1976b), "Distribution of tastes and skills, and the provision of local public goods," *Journal of Public Economics* 6, 406-423.

Brueckner, J.K., and S.S. Rosenthal, (2006), "Gentrification and Neighborhood Housing Cycles: Will America's future downtowns be rich?", mimeo.

Brueckner, J.K., J-F Thisse, and Y. Zenou, (1999), "Why is central Paris rich and downtown Detroit poor? An amenity based theory," *European Economic Review* 43, 91-107.

Cappoza, D. and R. W. Helsley, (1989), "The fundamentals of land prices and urban growth," *Journal of Urban Economics* 26, 295-306.

Cheshire, P., and S. Sheppard, (1998), "Estimating the demand for housing, land and neighbourhood characteristics," *Oxford Bulletin of Economics and Statistics* 60, 357-377.

de Bartolome, C.A.M., and S.L. Ross, (2003), "Equilibrium with local governments and commuting: income sorting and income mixing," *Journal of Urban Economics* 54, 1-20.de Bartolome and Ross 2004

de Bartolome, C.A.M, and S.L. Ross, (2004), "Who's in charge of the central city? The conflict between efficiency and equity in the design of a metropolitan area," *Journal of Urban*

Economics 56, 458-483.

de Bartolome, C.A.M. and S.L. Ross, (2007), "Community income distributions in a metropolitan area," forthcoming in *Journal of Urban Economics*.

Duncombe, W.D., (1991), "Demand for local public services revisted: the case of fire protection," *Public Finance Quarterly* 19 , 412-436

Dynarski, M., R. Schwab and E. Zampelli, (1989), "Local characteristics and public production: the case of education," *Journal of Urban Economics* 26 250-263.

Elickson, B., (1971), "Jurisdictional fragmentation and residential choice," *Papers and Proceedings of the American Economic Association* 61, 334-339.

Epple, D., and G.J. Platt, (1998), Equilibrium and local redistribution in an urban economy when households differ in both preferences and income," *Journal of Urban Economics*, 43, 23-51.

Epple, D., R. Filimon and T. Romer, (1984), "Equilibrium among local jurisdictions: toward an integrated treatment of voting and residential choice," *Journal of Public Economics* 24, 281-308.

Epple, D., R. Filimon and T. Romer, (1993), "Existence of voting and housing equilibrium in a system of communities with property taxes," *Regional Science and Urban Economics* 23, 585-610.

Glaeser, E.L., J. Gyourko and R. Saks, (2005), "Why is Manhatten so expensive? Regulation and the rise in housing prices," *Journal of Law and Economics* 48, 331-369.

Glaeser, E.L., M.E. Kahn and J. Rappaport, (2000), "Why do the poor live in the cities?" *National Bureau of Economic Research Working Paper* # 7636.

Gyourko, J., and R. Voith, (2001), "The price elasticity of the demand for residential land: estimation and some implications for urban form," *Working Paper of the Real Estate Department at Wharton*.

Inman, R.P., (1979), "The fiscal performance of local governments: an interpretive review," in *Current Issues in Urban Economics* (P. Miezkowski and M. Straszheim , eds.). Maltimore, MD: John Hopkins University Press.

King, A.T., (1976), "The demand for housing: a Lancastrian approach,"*Southern Economics Journal* , 43, 1077-1087.

LeRoy, S.F., and J. Sonstelie, (1983), "Paradise lost and regained: transportation, innovation, income and residential location," *Journal of Urban Economics* 13, 67-89.

McGuire, M.C., (1974), "Group segregation and optimal jurisdictions," *Journal of Political Economy* 82 ,112-132.

McMillan, M.L., (1979), "Estimates of households' preferences for environmental quality and other housing characteristics from a system of demand equations," *Scandinavian Journal of Economics*, 81, 174-187.

Mills, E.S., (1967). "An aggregative model of resource allocation in a metropolitan area,." *Papers and Proceedings of the American Economic Association* 57, 197-210.

Muth, R., (1961), "The spatial structure of the housing market," *Papers and Proceedings of the Regional Science Association* 7, 207-220.

Muth, R., (1969), *Cities and Housing*. Chicago: University of Chicago Press.

Muth, R., (1971), "The derived demand for urban residential land," *Urban Studies* 8, 243-254

Ross, S.L., and J. Yinger, (1999), "Sorting and voting: a review of the literature on urban public finance," in *The Handbook of Urban and Regional Economics, Vol 3: Applied Urban Economics* (P. Cheshire and E.S. Mills, eds.). Amsterdam: North Holland.

Schwab, R.M. and E.M. Zampelli, (1987), "Disentangling the demand function from the production function for local public services," *Journal of Public Economics* 33, 245-260.

Straszheim, M.R., (1975), *An econometric analysis of the urban housing market*. New York, NY: National Bureau of Economic Research.

Tiebout, C.M., (1956), "A pure theory of local expenditures," *Journal of Political Economy* 94, 416-424.

Wheaton, W., (1977), "Income and urban residence: An analysis of consumer demand for location," *American Economic Review* 67, 620-631.

Wheaton, W., (1982), "Urban residential growth under perfect foresight," *Journal of Urban Economics* 12, 1-12.

Wooders, M., (1978), "Equilibria, the core and jurisdictional structures in economies with a local public good," *Journal of Economic Theory* 18, 328-348.

Yinger, J., (1982), "Capitalization and the theory of local public finance," *Journal of Political Economy* 90, 917-943.

ENDNOTES

1. Ross and Yinger (1999) survey this literature.

2. In addition to considering the property tax, the model presented in this paper extends the earlier models by making house size endogenous.

3. We want to stress that, because each household's endowed income *M* is exogenous, its taste parameters $\alpha(M)$, $\beta(M)$ and $\gamma(M)$ are exogenous.

The reader should note that all households with the same endowed income have the same utility function so that "income-mixing" arises because of differences in tastes between income-classes and not because of differences in tastes between and among income classes (as in Epple and Platt (1998)).

4. For ease of calculation, the jurisdiction is assumed to provide a public service and not a public good. It is straightforward to change the publically-provided good from a public service to a public good.

5. Wheaton (1977) shows that, in the Alonso-Mills-Muth model, the bid-rent curve steepens with income if the income elasticity of land is less than the income elasticity of commuting cost. In our model the income elasticity of commuting cost is unity.

6. For diagrammatic clarity, the bid-rent schedules and the rent schedule are drawn as straight lines. In fact, as the location moves towards the city's center, the higher rents cause lot sizes to fall; this causes the bid-rent curves and the rent schedule steepen.

7. If only one income class resides in the jurisdiction, the bid-rent curve of the other income class lies below the bid-rent curve of the income class which resides in the jurisdiction.

8. In practice, if the incentive existed, a developer might build a development with houses targeted for households with his income so that he would move into the jurisdiction in the company of others with his income.

9. The assumption of myopia greatly simplifies the model but we do not believe any results depend on it. What is important is that the public service differs in the two jurisdictions. A similar assumption was made by Epple et al. (1984, 1993).

10. An algebraic formulation of the model is available from the authors on request.

11. Because utility is additively separable and linear in the numeraire, our results are unchanged if all rents are paid to absentee landlords.

12. As noted in Footnote 6, for diagrammatic clarity, the rent schedules are drawn as straight lines. In fact, as the location moves away from the metropolitan center, the slope of the rent schedule decreases.

13. Strictly, as the metropolitan population increases from N to $N + \Delta N$ provided there is a strict majority of one class in the suburb in the pre-existing equilibrium (at population N), this majority is maintained in the adjacent equilibrium at population ($N + \Delta N$).

14. Although the causation is quite different, this result resembles the leapfrog development pattern that may appear in models of urban growth where some land is left vacant in the interior because its option value for future development exceeds its value in current use. For some examples in the literature, see Arnott and Lewis (1979), Capozza and Helsley (1989) and Wheaton (1982).

15. With equal numbers of poor and rich households, the rich become the majority in the city immediately the poor spill over into the suburb.

16. Our continuity argument is strong in that it claims x, X, y and Y change continuously. A weaker argument is that, once a class establishes a majority in a jurisdiction, this majority is maintained provided that there is a (static) equilibrium with this majority.

17. The linear dependence on the numeraire implies that a household's desired level of the public service does not depend on his location. The logarithmic dependance of land implies that the rent schedule is relatively easy to calculate. The logarithmic dependance of housing capital implies that it is possible to obtain a closed-form solution for the tax rate. In addition, the linear and separable dependence on the numeraire implies that the rent returned does not affect the choice of l, h or g.

18. Our calibration assumes that there is a single city, that the public service is financed by a residence tax and that the public service is determined by rich or poor households with equal probability.

19. A list of all such possible equilibria is available from the authors on request.

20. The high price elasticity of land demand (relative to income elasticity) leads the average plot size of the rich to be less than that of the poor at metropolitan populations above 10 000 households.

21. We have simulated up to a metropolitan population of 100 million households.

22. If rich households in the "single city" were prevented from locating in the suburb, the city would become fully developed and poor households would spill into the suburb when $N = 403\ 934$.

23. The equilibrium has the form of Configuration E.

24. Brueckner, Thisse and Zenou (1999) attribute the European pattern to the large historical amenities present in many European cities (but absent from most U.S. cities).